

Math233 Calculus 3 Fall 09 Sample midterm 1

Name: _____

Problem 1. Let $\vec{u} = \langle 4, 4, 5 \rangle$ and $\vec{v} = \langle 2, -1, 1 \rangle$.

- (a) Find a unit vector in the direction of \vec{v} .
- (b) Find $\|\text{proj}_{\vec{v}}\vec{u}\|$.
- (c) Express \vec{u} as the sum of $\vec{m} = \vec{u}_{\parallel}$ parallel to \vec{v} , and $\vec{n} = \vec{u}_{\perp}$ orthogonal to \vec{v} .

Problem 2. Consider three points $A(-2, 1, -1)$, $B(1, 2, 2)$, $C(1, 1, 5)$.

- (a) Are the points A, B, C collinear? Justify your answer using the cross-product.
- (b) Find the area of the triangle formed by A, B, C .
- (c) Find the equation of the plane that contains A, B, C .

Problem 3. Consider two points $E(1, 0, 1)$, $F(-3, 2, 3)$.

- (a) Find a parametric equation of the line through E and F .
- (b) Find the symmetric equation of the line through E and F .
- (c) Find the cylindrical coordinates for E .
- (d) Find the spherical coordinates for E .

Problem 4.

- (a) Find the angle between the planes $x - y = 3$ and $-y + z = 1$, and the line of intersection.
(Hint: Angle between the planes is the angle between their normal vectors.)
- (b) Find the equation of the plane that passes through the point $(1, 2, -1)$ and is perpendicular to the line $x - 2 = \frac{y + 1}{2} = \frac{z}{4}$.

Problem 5. For each equation below, sketch the surface in \mathbf{R}^3 that it describes.

(a) $x^2 + 4y^2 + 4z^2 = 16$

(b) $4x^2 + y^2 + 4z^2 = 16$

(c) $z = 9x^2 + 4y^2$

(d) $z = 9x^2 - 4y^2$

(e) $9x^2 + 4y^2 = 2z^2 + 72$

(f) $9x^2 + 4z^2 = 2y^2 - 72$

(g) $9x^2 + 4y^2 = 2z^2$

(h) $9x^2 - 4y^2 = 72$