(1) Let \( \mathbf{v} \) be the vector \( (1, -1, 1) \), and let \( \mathbf{w} \) be the vector \( (3, 2, 2) \).

(a) Write \( \mathbf{w} \) as a sum of two vectors, one parallel to \( \mathbf{v} \), and one perpendicular to \( \mathbf{v} \).

(b) Find the equation of the plane through the origin which contains the two vectors \( \mathbf{v} \) and \( \mathbf{w} \).

a) find projection of \( \mathbf{w} \) onto \( \mathbf{v} \):

\[
\left( \mathbf{v}, \mathbf{w} \right) \frac{\mathbf{v}}{||\mathbf{v}||^2} = \frac{(3-2+2)}{3} \begin{bmatrix} 4 \ 1 \ 1 \end{bmatrix}
\]

so \( \mathbf{w} = \begin{bmatrix} 1 \ -1 \ 1 \end{bmatrix} + \begin{bmatrix} 2 \ 3 \ 1 \end{bmatrix} = \begin{bmatrix} 1 \ -1 \ 1 \end{bmatrix} \)

b) find a perpendicular vector:

\[
\mathbf{v} \times \mathbf{w} = \begin{vmatrix} 
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -1 & 1 \\
3 & 2 & 2
\end{vmatrix} = \begin{bmatrix} -4 \ 1 \ 5 \end{bmatrix}
\]

equation of plane: \(-4x + y + 5z = 0\)
(2) (a) What is the difference between speed and velocity?
(b) A particle starts at the origin at time zero, and has velocity given by
\[ r'(t) = (3 \sin(t), -4 \sin(t), 5 \cos(t)) \]. Where is the particle at time \( t = \pi \)?

a) velocity is a vector giving the rate of change of position with time. Speed is the length of the velocity vector.

b) \[ \mathbf{r}(t) = \langle -3 \cos(t), \ 4 \cos(t), \ 5 \sin(t) \rangle + c \]

\[ \mathbf{r}(0) = \langle -3, \ 4, \ 0 \rangle + c = \langle 0, 0, 0 \rangle \Rightarrow c = \langle 3, -4, 0 \rangle \]

\[ \mathbf{r}(\pi) = \langle -3 \cos(\pi), \ 4 \cos(\pi), \ 5 \sin(\pi) \rangle + \langle 3, -4, 0 \rangle \]

\[ = \langle 3, -4, 0 \rangle + \langle 3, -4, 0 \rangle = \langle 6, -8, 0 \rangle. \]
(3) (a) Define the gradient vector and describe its geometric properties.
(b) Find the gradient vector of the function \( f(x, y) = x^2 \sin(3x + 2y) \) at the point \((2, -3)\).

\[ \nabla f(x, y) = \left< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right> \]

\[ \nabla f(2, -3) = \left< 4 \sin(0) + 12 \cos(0), 8 \cos(0) \right> = \left< 12, 8 \right> \]

a) The gradient vector points in the direction of fastest rate of change, and its length is the fastest rate of change.

b) \( \nabla f = \left< 2x \sin(3x+2y) + x^2 \cos(3x+2y), 2x \cos(3x+2y) \right> \)
(4) Find all second partial derivatives of the function $f(x, y) = x^2 - y^3 + 4x + 6y^2 - 10$.

\[ f_x = 2x + 4 \]
\[ f_y = -3y^2 + 12y \]
\[ f_{xx} = 2 \]
\[ f_{xy} = 0 \]
\[ f_{yy} = -6y + 12 \]
(5) Find the critical points of the function \( f(x, y) = x^2 - y^3 + 4x + 6y^2 - 10 \) and use the second derivative test to classify them, if possible. Feel free to use your answer to the previous question.

Solve \[ \begin{cases} f_x = 0 & \Rightarrow 2x + 4 = 0 \quad x = -2 \\ f_y = 0 & \Rightarrow -3y^2 + 12y = 0 \quad 3y(-y + 4) = 0 \quad y = 0, 4 \end{cases} \]

Two critical points \((-2, 0)\) and \((-2, 4)\).

Second derivative test: \( D(x, y) = f_{xx}f_{yy} - (f_{xy})^2 \)

\[ D(-2, 0) = 2(12) - 0^2 = 24 > 0 \quad f_{xx} = 2 > 0 \quad \text{so local min.} \]

\[ D(-2, 4) = 2(-12) - 0^2 = -24 < 0 \quad \text{so saddle point}. \]
(6) Evaluate the following double integral by changing the order of integration.

\[
\int_0^\sqrt{\pi} \int_y^\sqrt{\pi} \sin\left(\frac{1}{2}x^2\right) dx \, dy
\]

\[
\int_0^{\sqrt{\pi}} \int_0^x \sin\left(\frac{1}{2}x^2\right) dy \, dx
\]

\[
\left[ y \sin\left(\frac{1}{2}x^2\right) \right]_0^x = x \sin\left(\frac{1}{2}x^2\right)
\]

\[
\int_0^{\sqrt{\pi}} x \sin\left(\frac{1}{2}x^2\right) dx = \left[ -\cos\left(\frac{1}{2}x^2\right) \right]_0^{\sqrt{\pi}}
\]

\[
= -\cos\left(\frac{\pi}{2}\right) + \cos(0) = 1
\]
(7) Write down a triple integral which gives the integral of the function \( f(x, y, z) = xyz \) over the region in the positive octant underneath the plane \( x + y + 2z = 6 \) using a triple integral. Do not evaluate this integral.
Write down a triple integral which gives the integral of the function \( f(x, y, z) = x + y + z \) over the ice cream cone shaped region above the positive cone \( z = \sqrt{x^2 + y^2} \) and below \( x^2 + y^2 + z^2 = 1 \). Do not evaluate this integral.

Use spherical coordinates:

\[
\begin{align*}
    x &= \rho \sin \phi \cos \theta \\
    y &= \rho \sin \phi \sin \theta \\
    z &= \rho \cos \phi \\
    \rho^2 &= x^2 + y^2 \\
    \phi &= \pi / 4 \\
    \sin \phi &= 1 \\
\end{align*}
\]

\[
\int_0^1 \int_0^{2\pi} \int_0^{\pi/4} (\rho \sin \phi (\cos \theta + \sin \theta) + \rho \cos \phi) \rho^2 \sin \phi \ d\phi \ d\theta \ d\rho
\]