Math 233 Calculus 3 Fall 09 Sample Final

Name: ____________________________

(1) (a) Find a vector $\mathbf{n}$ perpendicular to both the line $x = 1+t, y = 1+t, z = 1-t$ and the x-axis.
(b) Write down the equation of the plane through the point $(2, 2, 2)$ which is perpendicular to $\mathbf{n}$.

(2) Using the plane from the previous question, find the distance from the plane to the origin, by
(a) using the dot product.
(b) by setting up and solving a minimization problem.

(3) Let $\mathbf{a} = \langle -2, 4, 2\rangle$ and $\mathbf{b} = \langle 1, 0, -1\rangle$. Express $\mathbf{a}$ as a sum of vectors $\mathbf{x} + \mathbf{y}$, where $\mathbf{x}$ is parallel to $\mathbf{b}$, and $\mathbf{y}$ is perpendicular to $\mathbf{b}$.

(4) Find the arc length of the curve

$$x = \cos t + t \sin t, y = \sin t - t \cos t, z = t^2$$

from $t = 0$ to $t = \pi/2$.

(5) A particle located at $\langle 7, 5, 0\rangle$ at $t = 0$ has velocity $\mathbf{v} = \langle 1, 3t^2, 2t^3\rangle$. Find the location of the particle at $t = 2$.

(6) The power P used by my refrigerator is equal to its voltage V times the current I, and the voltage is equal to its resistance R times the current. The mains voltage is roughly 110 volts, and the fridge averages roughly 200 watts. I want to estimate the power used, and I can measure any two of voltage, resistance or current to an accuracy of 10%, which two should I pick to get the most accurate estimate for power, or does it not matter?

(7) Consider the function of three variables defined by

$$f(x, y, z) = e^{2x-y} + \tan(yz).$$

(a) What is the direction of fastest rate of change of the function at the point $(1, 2, 0)$?
(b) What is the tangent plane to the surface $e^{2x-y} + \tan(yz) = 1$ at the point $(1, 2, 0)$?

(8) Find the critical points of the function $f(x, y) = 3xy - x^2y - xy^2$. Use the second derivative test to classify them, if possible.

(9) A rectangular/cuboid building loses twice as much heat per unit area from the walls than from the roof and floor. What shape should the building be to mimize heat loss if the volume is 4000m^3?

(10) Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$$

Hint: change the order of integration.

(11) Find the center of mass of the region above the x-axis which lies between the curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, with density funtion $\rho(x, y) = x^2 + y^2$. You may assume that $\overline{x} = 0$ by symmetry.

(12) Evaluate the following integral

$$\int\int\int_E dV$$

where E is the region in the first octant bounded by the planes $x = 2$ and $x + y + z = 4$.

(13) Find the volume of the region inside the sphere $x^2 + y^2 + z^2 = 9$ which lies above the plane $z = 0$ and below the positive cone $z = \sqrt{x^2 + y^2}$.

(14) Evaluate the following integral

$$\int\int\int_E xyz\ dV$$

where E lies between the spheres $\rho = 2$ and $\rho = 4$, and above the cone $\phi = \pi/3$.