# Linear progress in the complex of curves

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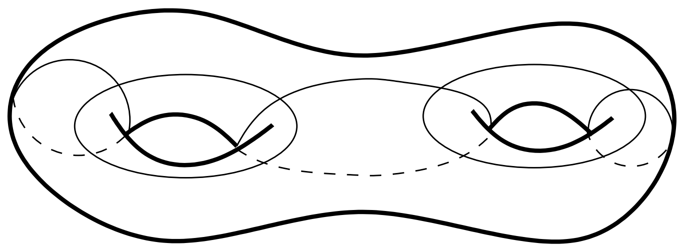
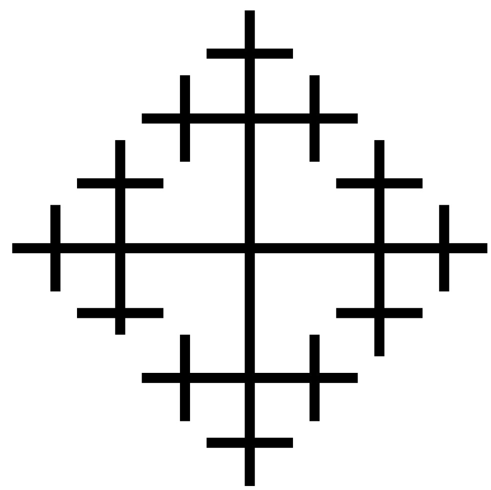
$\Sigma$ closed orientable surface



Def: $G = \mathrm{MCG}(\Sigma)$
$= \mathrm{Diff}^+(\Sigma)/\mathrm{Diff}_0(\Sigma)$



Finitely generated

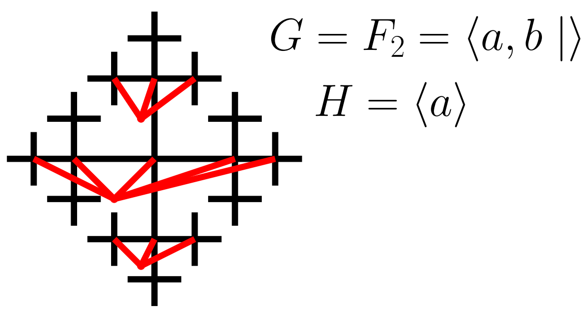
Let $\Gamma$ be a Cayley graph for $G$, consider nearest neighbour random walk on $\Gamma$

[Kesten et al.] $|w_n|_\Gamma$ grows linearly.

i.e. $\mathbb{P}(\frac{n}{E} \leqslant |w_n|_\Gamma \leqslant nE) \to 1$, as $n \to \infty$.

[Masur-Minsky] $G$ is weakly relatively hyperbolic.

Given $H < G$, the relative space $\widehat{\Gamma}$ is $\Gamma$, with each coset $gH$ coned off to a vertex $v_{gH}$.




If $\widehat{\Gamma}$ is $\delta$-hyperbolic then we say $G$ is weakly relatively hyperbolic

[Masur-Minsky] $\widehat{\Gamma} \sim_{QI} \mathcal{C}(\Sigma)$ complex of curves.

[M] $|w_n|_{\widehat{\Gamma}}$ grows linearly

[Klarreich] $\partial\widehat{\Gamma}$ = minimal foliations $\subset \mathcal{PML}$

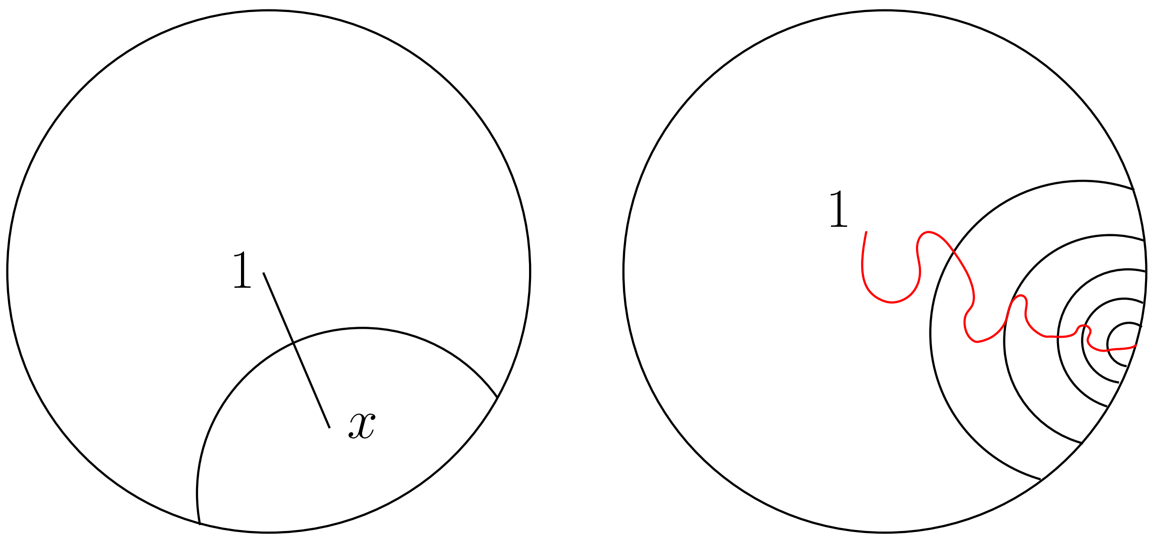
[Kaimanovich-Masur] $w_n \to \lambda \in \mathcal{PML}$ a.s.

This gives a measure $\nu$ on $\mathcal{PML}$,
$\nu(X)$ = probability $w_n$ converges to $\lambda \in X$

Half space $H(1, x) = \{y \in \widehat{\Gamma} \mid \widehat{d}(x, y) \leqslant \widehat{d}(y, 1)\}$

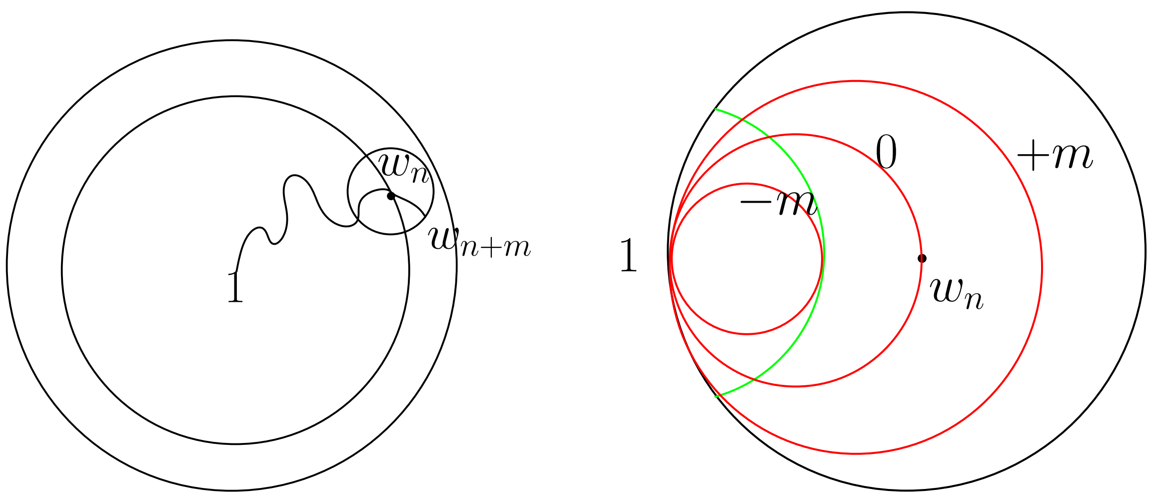
Lemma: $\nu(H(1,x)) \leqslant L^{|x|_{\widehat{\Gamma}}}$, for some constant $L < 1$ independent of $x$

Proof: $\nu(H(1,x)) \leqslant 1 - \epsilon$ for all $|x|_{\widehat{\Gamma}} \geqslant K$, for constants $\epsilon > 0$ and $K$




Nested half spaces, conditional probability.

Linear progress:




For large enough $m$, $-mL^m + (1 - L) \geqslant \epsilon > 0$

So $\mathbb{E}(w_{n+m}) \geqslant \mathbb{E}(w_n) + \epsilon$

use: Kingman's subadditive ergodic theorem