Asymptotics for pseudo-Anosovs in Teichmüller lattices

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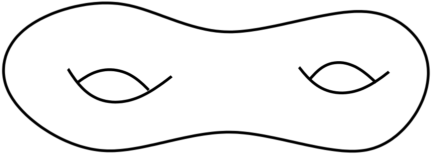
S closed orientable surface



Def: $G = \mathrm{MCG}(S) = \mathrm{Homeo}^+(S)/\text{isotopy}$

[Thurston] Classification of elements of G:

- Periodic
- Reducible
- Pseudo-Anosov

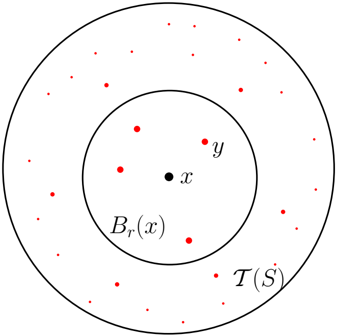
Teichmüller space $\mathcal{T}(S) \cong \mathbb{R}^{6g-6}$

- Space of conformal structures on S
- Space of hyperbolic structures on S

Teichmüller metric: $d_{\mathcal{T}}(x, y) = \inf \frac{1}{2} \log K$

- $(\mathcal{T}, d_{\mathcal{T}})$ infinite diameter, complete
- G acts by isometries on $\mathcal{T}$, properly discontinously
- unique geodesic connecting any pair of points
- moduli space $\mathcal{T}/G$ finite volume

Teichmüller lattice: Gy



[Athreya, Bufetov, Eskin, Mirzakhani]

$$|Gy \cap B_r(x)| \sim C(x, y)e^{hr}$$

cf [Margulis]

Def: R = non-pseudo-Anosov elements of G.

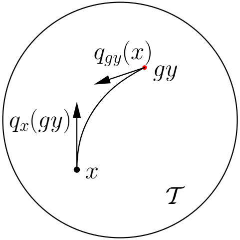
Thm[M]:

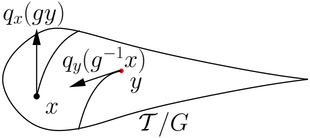
$$\frac{|Ry \cap B_r(x)|}{|Gy \cap B_r(x)|} \to 0 \text{ as } r \to \infty.$$

Q = unit area quadratic differentials = "unit tangent bundle of $\mathcal{T}$"

$g_t : Q \to Q$ geodesic flow

$\pi : Q \to \mathcal{T}$, $S(x) = \pi^{-1}(x)$ = visual boundary





bisector: $U \subset S(x), V \subset S(y)$

$g \in B(U,V) \iff q_x(gy) \in U$ and $q_y(\gamma^{-1}x) \in V$

Thm[ABEM]:

$$|Gy \cap B_r(x), g \in B(U,V)| \sim \frac{1}{h} e^{hr} \Lambda^+(U) \Lambda^-(V)$$

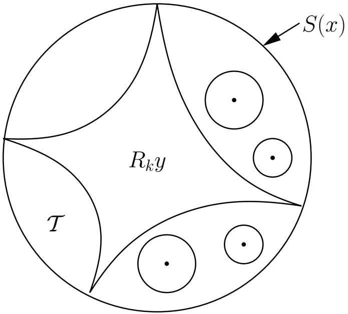
Λ^+, Λ^- measures on $S(x), S(y)$ respectively, defined in terms of the Masur-Veech measure μ on Q, which is g_t-invariant, with $\mu(Q/G) = 1$.

Note: distribution of leaving directions $q_x(gy)$ given by Λ^+, distribution of arriving directions $q_y(g^{-1}x)$ given by Λ^-, independent.

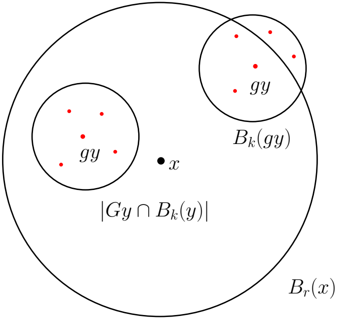
Consider R = set of non-pseudo-Anosov elements.

$R_k = \{g \in R \mid d_{\mathcal{T}}(gy, g'y) \leqslant k, \text{ some } g' \in R \setminus g\}$ "k-dense"

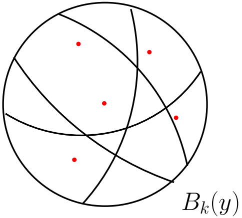
$R \setminus R_k$ "k-separated"



Thm[M]: $\overline{R_k}$ has measure zero in visual boundary



Equidistribution:



Thm[Veech]: The Teichmüller geodesic flow is mixing.

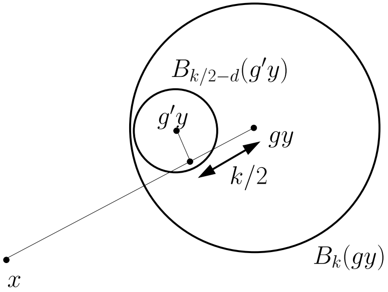
$$\lim_{t \to \infty} \int_{Q/G} \alpha(g_t q) \beta(q) d\mu(q) = \int_{Q/G} \alpha(q) d\mu(q) \int_{Q/G} \beta(q) d\mu(q)$$

Conditional mixing:

$$\lim_{t \to \infty} \int_{S(x)} \alpha(g_t q) \beta(q) ds_x(q) = \int_{Q/G} \alpha(q) d\mu(q) \int_{S(x)} \beta(q) ds_x(q)$$

Here α, β continuous, compact support.

Go back distance $k/2$ along geodesic from x to gy, look for lattice point distance at most $d < k/2$ away, get at least $|Gy \cap B_{k/2-d}(y)|$ lattice points in $B_k(gy) \cap B_r(x)$.



i.e. this estimate works for the proportion of lattice points in $\partial B_{k/2}(y)$ which lie in $N_d(Gx)$, mixing implies this is $\mathrm{vol}(N_d(x))$ in Q/G, tends to 1 as $d \to \infty$.