Asymptotics for pseudo-Anosovs in Teichmüller lattices

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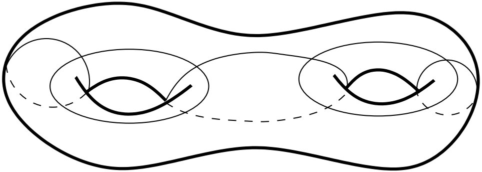
Σ closed orientable surface



Def: $\Gamma = \mathrm{MCG}(\Sigma) = \mathrm{Diff}^+(\Sigma)/\mathrm{Diff}_0(\Sigma)$

[Thurston] Classification of elements of Γ:

- Periodic
- Reducible
- Pseudo-Anosov

Teichmüller space $\mathcal{T}(\Sigma) \cong \mathbb{R}^{6g-6}$:

- Space of conformal structures on Σ
- Space of hyperbolic structures on Σ

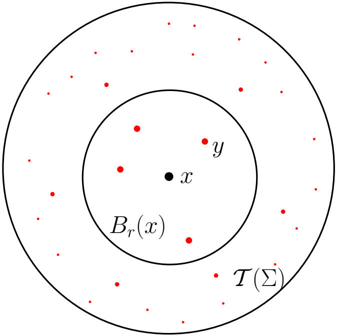
Teichmüller metric: $d(x, y) = \inf \frac{1}{2} \log K$

infimum over $f : x \to y$ is K-quasiconformal

Γ acts by isometries on $\mathcal{T}$

moduli space $\mathcal{T}/\Gamma$ finite volume

Teichmüller lattice: Γy



[Athreya, Bufetov, Eskin, Mirzakhani]

$$|\Gamma y \cap B_r(x)| \sim C(x, y) e^{hr}$$

cf [Margulis] [Sharpe's survey article]

Def: R = non-pseudo-Anosov elements of Γ.

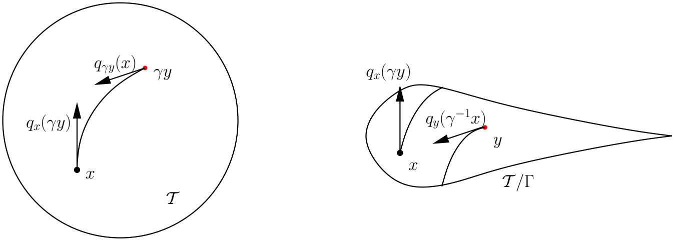
Thm[M]:

$$\frac{|Ry \cap B_r(x)|}{|\Gamma y \cap B_r(x)|} \to 0 \text{ as } r \to \infty.$$

Q = unit area quadratic differentials = "unit tangent bundle of $\mathcal{T}$"

$g_t : Q \to Q$ geodesic flow

$\pi : Q \to \mathcal{T}$, $S(x) = \pi^{-1}(x)$ = visual boundary



bisector: $U \subset S(x), V \subset S(y)$

$\gamma \in B(U, V) \iff q_x(\gamma y) \in U$ and $q_y(\gamma^{-1}x) \in V$

Thm[ABEM]:

$$|\Gamma y \cap B_r(x), \gamma \in B(U,V)| \sim \frac{1}{h} e^{hr} \Lambda_x^+(U) \Lambda_y^-(V)$$

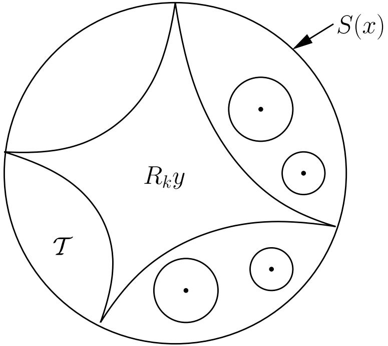
Λ_x^+, Λ_y^- measures on $S(x), S(y)$ respectively, defined in terms of the Masur-Veech measure μ on Q, which is g_t-invariant, with $\mu(Q/\Gamma) = 1$.

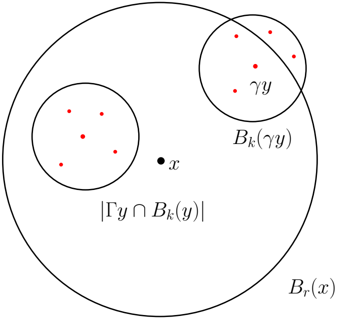
Note: distribution of leaving directions $q_x(\gamma y)$ given by Λ^+, distribution of arriving directions $q_y(\gamma^{-1} x)$ given by Λ^-, independent.

Consider $R =$ set of non-pseudo-Anosov elements.

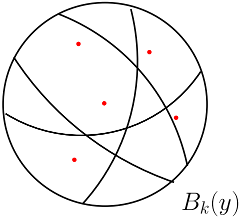
$R_k = \{\gamma \in R \mid d_{\mathcal{T}}(\gamma y, \gamma' y) \leqslant k, \text{ some } \gamma' \in R \setminus \gamma\}$

Thm[M]: $\overline{R_k}$ has measure zero in visual boundary





Equidistribution:



Thm[Veech]: The Teichmüller geodesic flow is mixing.

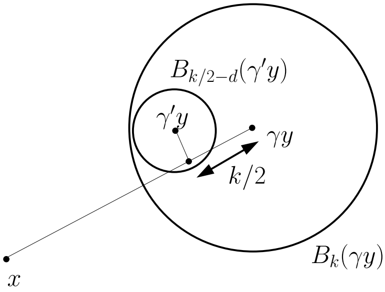
$$\lim_{t\to\infty}\int_{Q/\Gamma}\alpha(g_t q)\beta(q)d\mu(q) = \int_{Q/\Gamma}\alpha(q)d\mu(q)\int_{Q/\Gamma}\beta(q)d\mu(q)$$

Conditional mixing:

$$\lim_{t\to\infty}\int_{S(x)}\alpha(g_t q)\beta(q)ds_x(q) = \int_{Q/\Gamma}\alpha(q)d\mu(q)\int_{S(x)}\beta(q)ds_x(q)$$

Here α,β continuous, compact support.

Go back distance $k/2$ along geodesic from x to γy, look for lattice point distance at most $d < k/2$ away, get at least $|\Gamma y \cap B_{k/2-d}(y)|$ lattice points in $B_k(\gamma y) \cap B_r(x)$.



i.e. this estimate works for the proportion of lattice points in $\partial B_{k/2}(y)$ which lie in $N_d(\Gamma x)$, mixing implies this is $\mathrm{vol}(N_d(x))$ in Q/Γ, tends to 1 as $d \to \infty$.