Spring 2005

Simplifying Rational Expressions

A rational expression has a numerator and a denominator. Sometimes these are polynomials of the form $ax^2 + bx + c$, or sometimes difference of squares $(ax)^2 - b^2$. The strategy for solving such expressions involves factoring and then canceling:

$$\text{consider: } \frac{x^2 - 3x - 10}{25 - x^2}$$

First, factor the numerator: $x^2 - 3x - 10 = (x-5)(x+3)$
Then factor the denominator: $25 - x^2 = 5^2 - x^2 = (5+x)(5-x)$

$$\text{the result is } \frac{x^2 - 3x - 10}{25 - x^2} = \frac{(x-5)(x+2)}{(5+x)(5-x)}$$

now we just have to cancel: $\big(\text{note that } (5-x) = -(x-5)\big)$

$$\frac{(x-5)(x+2)}{(5+x)(5-x)} = \frac{(x-5)(x+2)}{-(x-5)(5+x)} = \frac{\cancel{(x-5)}(x+2)}{-\cancel{(x-5)}(x+5)} = \frac{x+2}{-(x+5)} = -\frac{x+2}{x+5}$$

Multiply and Divide Rational Expressions

Multiplying or dividing rational expressions involves first simplifying and canceling, as before:

1.
$$\text{multiply: } \frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^2}$$

$$\frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^2} \quad = \quad \frac{8x^2}{4x^2} \cdot \frac{3y^2}{9y^3} \quad = \quad \frac{\cancel{4} \cdot 2x^2}{\cancel{4}x^2} \cdot \frac{\cancel{3}y^2}{\cancel{3} \cdot 3y^3} \quad = \quad \frac{2x^2}{x^2} \cdot \frac{y^2}{3y^3} = \frac{2}{3y}$$

2.
$$\text{divide: } \frac{6x - 12}{8x + 32} \div \frac{18x - 36}{10x + 40}$$

First, change the operation from division to multiplication by taking the reciprocal as shown below:

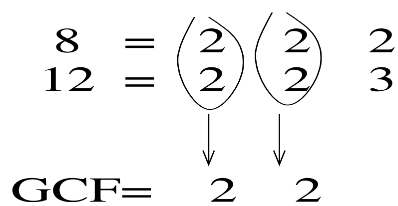
$$\frac{6x - 12}{8x + 32} \div \frac{18x - 36}{10x + 40} = \frac{6x - 12}{8x + 32} \cdot \frac{10x + 40}{18x - 36}$$

Now just factor and cancel as before:

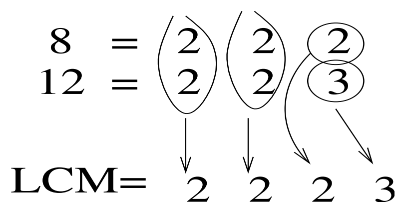
$$\frac{6x - 12}{8x + 32} \cdot \frac{10x + 40}{18x - 36} = \frac{6(x-2)}{8(x+4)} \cdot \frac{10(x+4)}{18(x-2)} = \frac{6}{8} \cdot \frac{10}{18} = \frac{3}{4} \cdot \frac{5}{9} = \frac{3}{4} \cdot \frac{5}{3^2} = \frac{5}{12}$$

Least Common Multiple (LCM) of Polynomials

First Review the GCF: Recall that the Greatest Common Factor, or GCF, of two expressions is the largest expression that divides evenly into both expressions:
example: the GCF of $8x^2y^4$ and $12x^3y^2$ is $4x^2y^2$, because $4x^2y^2$ is the largest expression that divides into both $8x^2y^4$ and $12x^3y^2$ without a remainder. We use the GCF when we want to factor an expression such as $8x^2y^4 + 12x^3y^2$.
The GCF of 8 and 12 is found by factoring 8 and 12 into its prime factors:



the GCF is $2 \cdot 2 = 4$ since both 8 and 12 have $2 \cdot 2$ or 4.
Now the least Common multiple, of LCM, of two or more expressions is found in a similar manner. Let's find the LCM of 8 and 12:



We start by pairing common factors, and then also including the factors that are unique to each. Thus, the **LCM** of 8 and 12 is $2 \cdot 2 \cdot 2 \cdot 3 = 24$.
So we use the GCF when factoring. Also, we use the LCM when adding or subtracting Rational Expressions:

$$\text{Simplify} \quad \frac{3}{8x^2y^4} + \frac{5}{12x^3y^2} \tag{1}$$

The LCM of $8x^2y^4$ and $12x^3y^2$ must first be found. First, as we've found above, the LCM of 8 and 12 is 24. Now find the LCM of x^2 and x^3. This is the opposite of finding the GCF. The GCF of x^2 and x^3 would be x^2. However, the LCM is x^3, the variable with the higher numbered exponent. Similarly, the LCM of y^4 and y^2 is y^4. Thus, the LCM of the two denominators would be $24x^3y^4$
So the expression can be added as follows:

$$\frac{3(3x)}{24x^3y^4} + \frac{5(2y^2)}{24x^3y^4} = \frac{9x + 10y^2}{24x^3y^4} \tag{2}$$