Sample problems for Exam 3 for Math 233

This sample exam has many more questions than the actual exam will have.

1. Show that the following limits do not exist:
$$\lim_{(x,y)\to(0,0)} \frac{xy + xy^2}{x^2 + y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{(xy)^2}{x^4 + y^4}$$

2. Compute $\nabla f(1,2)$ for the following functions:
$$f(x,y) = 4xy^3 \qquad f(x,y) = \ln(x^2 + xy^2)$$

3. Find the total differential for the following functions:
$$f(x,y) = x\cos\left(\frac{y}{x}\right) \qquad f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

4. Find an equation of the tangent plane to the graph of $f(x,y) = xy^2 - xy + 3x^3y$ at $P(1,3)$.

5. Suppose the plane $z = 2x - y - 3$ is tangent to the graph of $z = f(x,y)$ at $P(2,4)$.
Find $f(2,4),\ f_x(2,4),\ f_y(2,4)$.
Approximate $f(2.2,\ 3.9)$.

6. Compute the directional derivative at P in the direction $\mathbf{v}$ for the following functions:
$$f(x,y,z) = zx - xy^2, \qquad P(3,-1), \qquad \mathbf{v} = (2,-1,2)$$
$$f(x,y,z) = \sin(xy+z), \qquad P(0,0,0), \qquad \mathbf{v} = \mathbf{j} + \mathbf{k}$$

7. Find an equation of the tangent plane at $P(0,3,-1)$ to the surface
$$ze^x + e^{z+1} = xy + y - 3.$$

8. Let $f(x,y) = x^2y + y^2z$. If $x = s+t,\ y = st,\ z = 2s - t$, compute $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

9. Find the critical points and analyze them using the Second Derivate Test for the following functions:
$$f(x,y) = x^2 + 2y^2 - 4xy + 6x \qquad f(x,y) = x^3 + 2y^3 - xy$$

10. Use Lagrange multipliers to find the minimum and maximum value of $f(x,y) = 3x - 2y$ on the circle $x^2 + y^2 = 4$.

11. Use Lagrange multipliers to find the minimum and maximum value of $f(x,y) = x^2y$ on the ellipse $4x^2 + 9y^2 = 36$.

12. Use Lagrange multipliers to find the dimensions of a cylindrical can of fixed volume $V = 1$ with minimal surface area, including the top and bottom of the can.

13. Find the dimensions of the box of maximum volume that can be placed inside the ellipsoid
$$(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$$