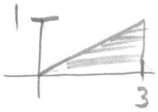
Mth 233 Exam 2 11/2/2016 Solutions

① $\int_{-1}^{1}\int_{2x^2}^{1+x^2} x^2\,dy\,dx = \int_{-1}^{1} x^2[(1+x^2)-(2x^2)]\,dx = \int_{-1}^{1} x^2-x^4\,dx = \frac{4}{15}$

② 

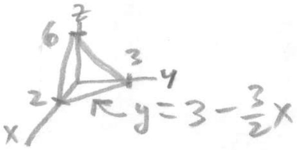
$\int_0^3\int_0^x f(x,y)\,dy\,dx + \int_3^4\int_0^{4x-x^2} f(x,y)\,dy\,dx = \int_0^3\int_y^{2+\sqrt{4-y}} f(x,y)\,dx\,dy$

$y=4x-x^2 \Rightarrow x^2-4x+4=4-y \Rightarrow (x-2)^2=4-y \Rightarrow x=2\pm\sqrt{4-y}$

③ 

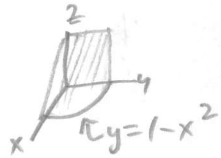
$\int_0^1\int_{3y}^{3} e^{x^2}\,dx\,dy = \int_0^3\int_0^{x/3} e^{x^2}\,dy\,dx = \int_0^3 \frac{1}{3}xe^{x^2}\,dx = \int_0^9 \frac{1}{6}e^u\,du$

$x=3y \Rightarrow y=x/3$ $\qquad u=x^2,\ du=2x\,dx$ $\qquad =\frac{1}{6}(e^9-1)$

④ 

$V=\int_0^2\int_0^{3-\frac{3}{2}x}\int_0^{6-3x-2y} dz\,dy\,dx = \int_0^2\int_0^{3-\frac{3}{2}x}(6-3x-2y)\,dy\,dx$

$=\int_0^2 (6-3x)(3-\frac{3}{2}x)-(3-\frac{3}{2}x)^2\,dx = \int_0^2 9-9x+\frac{9}{4}x^2 = 6$

⑤ 

$\iiint_E x\,dV = \int_0^1\int_0^{1-x^2}\int_0^{1-x} x\,dz\,dy\,dx = \int_0^1\int_0^{1-x^2} x(1-x)\,dy\,dx = \int_0^1 x(1-x)(1-x^2)\,dx$

$=\int_0^1 x-x^3-x^2+x^4 = \frac{1}{2}-\frac{1}{4}-\frac{1}{3}+\frac{1}{5} = \frac{7}{60}$

⑥ $f(x,y)=3x^3+y^2-9x-6y+1$

$\nabla f=(9x^2-9,\ 2y-6)=0 \Rightarrow (x,y)=(\pm1,\ 3)$ C.P.

$D=\begin{vmatrix}18x & 0\\ 0 & 2\end{vmatrix}=36x \Rightarrow D(-1,3)=-36<0 \Rightarrow (-1,3)$ saddle pt.

$D(1,3)=36>0,\ f_{xx}=18>0 \Rightarrow (1,3)$ Rel. <u>min</u>

⑦ $T(x,y)=2x+4y+6z, \quad g(x,y,z)=x^2+y^2+z^2=14$

$\nabla T=\lambda\nabla g \Rightarrow (2,4,6)=\lambda(2x,2y,2z)$

$2=\lambda\cdot 2x \Rightarrow x=1/\lambda$
$4=\lambda\cdot 2y \Rightarrow y=2/\lambda$
$6=\lambda\cdot 2z \Rightarrow z=3/\lambda$

$x^2+y^2+z^2=14 \Rightarrow (\frac{1}{\lambda})^2+(\frac{2}{\lambda})^2+(\frac{3}{\lambda})^2=14 \Rightarrow 14\lambda^2=14 \Rightarrow \lambda=\pm1$

CP: $(1,2,3)$ and $(-1,-2,-3)$. $\quad T(1,2,3)=28,\ T(-1,-2,-3)=\underline{-28}$

⑧ $f(x,y)=3x^2+4y^2-6x-5 \quad \nabla f=\lambda\nabla g \Rightarrow (6x-6,\ 8y)=\lambda(2x,2y) \Rightarrow$ $6x-6=\lambda\cdot 2x$ (1), $8y=\lambda\cdot 2y$ (2)

(2) $\Rightarrow \lambda=4$ or $y=0$. If $y=0$, $x=\pm4$. If $\lambda=4$, (1) $\Rightarrow x=-3 \Rightarrow y^2=7$

a) C.P. on $x^2+y^2=16$ are $(\pm4,0)$, $(-3,\pm\sqrt{7})$

b) $\nabla f=(6x-6,\ 8y) \overset{set}{=} 0 \Rightarrow$ C.P. $(1,0)$

$f(\pm4,0)=48\mp24-5=19,\ 67$

$f(-3,\pm\sqrt{7})=68$ <u>Max</u>

$f(1,0)=-8$ <u>Min</u>

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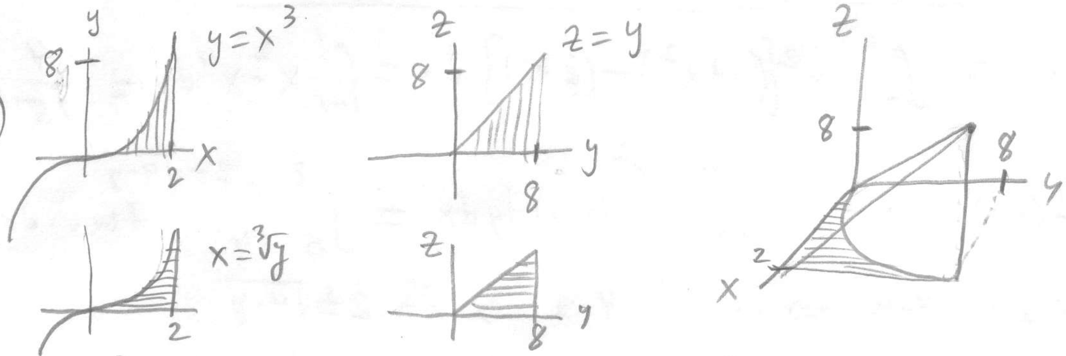

$$\int_0^2 \int_0^{x^3} \int_0^y f \, dz\, dy\, dx = \int_0^8 \int_z^8 \int_{\sqrt[3]{y}}^2 f \, dx\, dy\, dz$$