Calculus III (Math 233) Exam 2

November 2, 2016 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: _____

Problem 1. Evaluate $\iint_D x^2 \, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Problem 2. Let D be the region bounded by $y = x$, $y = 0$ and $y = 4x - x^2$. (Note that D is below the line $y = x$.) Write $\iint_D f(x, y) \, dA$ as an iterated integral in two different ways: in the order $dx\, dy$ and $dy\, dx$.

Problem 3. Change the order of integration to evaluate $\int_0^1 \int_{3y}^3 e^{(x^2)} \, dx\, dy$.

Problem 4. Using an integral, compute the volume of the tetrahedron bounded by the plane $3x + 2y + z = 6$ in the first octant.

Problem 5. Evaluate $\iiint_E x \, dV$ where E is the solid in the first octant ($x, y, z \geq 0$) which is bounded by the surface $y = 1 - x^2$ and the plane $z = 1 - x$.

Problem 6. Find all the critical points of $f(x, y) = 3x^3 + y^2 - 9x - 6y + 1$, and classify them using the Second Derivative Test.

Problem 7. Suppose the temperature on the sphere $x^2 + y^2 + z^2 = 14$ is given by $T(x, y, z) = 2x + 4y + 6z$. Find the temperature at the coldest point on the sphere.

Problem 8.

(a) Find the max and min of $f(x, y) = 3x^2 + 4y^2 - 6x - 5$ on the circle $x^2 + y^2 = 16$.

(b) Find the max and min of $f(x, y) = 3x^2 + 4y^2 - 6x - 5$ on the disc $x^2 + y^2 \leq 16$. (Use your answer in part (a).)

Problem 9. Change the order of integration to $dx\, dy\, dz$: $\int_0^2 \int_0^{x^3} \int_0^y f(x, y, z) \, dz\, dy\, dx$