Calculus III (Math 233) Exam 1

Date: September 29, 2010 $\qquad$ Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: _____

Problem 1. Find the equation of a plane containing the line $\ell(t) = (2+3t, \, -t, \, 4+t)$ and passing through the point $P(0, 2, -1)$.

Problem 2. The position of a particle is $\mathbf{r}(t) = (3t, \sin 4t, \cos 4t)$.

(a) Find the speed of the particle $v(t)$.

(b) Find the unit tangent vector $\mathbf{T}(t)$.

(c) Find the unit normal vector $\mathbf{N}(t)$, and verify that $\mathbf{T}(t) \perp \mathbf{N}(t)$.

(d) Find the curvature $\kappa(t)$.

(e) Find the arclength for $\pi \leq t \leq 2\pi$.

Problem 3. Let S be the surface $x^2 + 4y^2 - z^2 = 9$.

(a) Sketch the three traces of S, and then sketch S.

(b) Find the equation of the tangent plane to S at the point $P(-3, 2, 4)$.

Problem 4.

(a) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2x^4 + y^4}$ does not exist.

(b) Let $h(x, y) = x \sin(x + 2y)$. Verify Clairaut's Theorem: $h_{xy} = h_{yx}$.

Problem 5. Assuming the earth is a round sphere, show that when you drive around in a car, your velocity vector is always tangent to the earth.