Solutions to Sample problems for Final for Math 233

- Please attempt the problems before looking at the solutions.
- Please email me if you find any mistakes or typos.

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1. (a) $\int_0^1 \int_0^{1-y^2} xy + 2x + 3y \, dx \, dy = 41/30$

   (b) $\int_0^1 \int_0^{x^2} xe^y \, dy \, dx = (e-2)/2$

   (c) $\int_{-2}^2 \int_{x^2-3}^{5-x^2} xy \, dy \, dx = 0$

2. (a) $\int_0^{\pi/3} \int_0^3 r^4 \, dr \, d\theta = 81\pi/5$

   (b) $\int_{-\pi/2}^{\pi/2} \int_0^2 r^3 \, dr \, d\theta = 4\pi$

3. $\int_0^{1/2} \int_0^{2x} e^{-x^2} dy \, dx = \int_0^{1/2} 2x \, e^{-x^2} \, dx = 1 - e^{-1/4}$

4. $\int_0^{\pi/2} \int_0^4 \sqrt{1+4r^2} \, r \, dr \, d\theta = (\pi/24)(65^{3/2} - 1)$

5. (a) $\int_0^2 \int_0^{2x} \int_0^x x^2 \, dz \, dy \, dx = 64/5$

   (b) $\int_0^1 \int_0^{2-2x} \int_0^{2-y-2x} y \, dz \, dy \, dx = 1/3$

6. (a) $\int_0^{2\pi} \int_0^1 \int_0^{1-r^2} r^3 \, dz \, dr \, d\theta = \pi/6$

   (b) $\int_0^{\pi/2} \int_0^{2\pi} \int_0^1 \rho^8 \cos^5\phi \sin\phi \, d\rho \, d\theta \, d\phi = \pi/27$

7. (a) Cylindrical. $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{64-4r^2}}^{\sqrt{64-4r^2}} r \, dr \, d\theta$

   (b) Cylindrical. $\int_0^{\pi/4} \int_0^1 \int_0^1 r \, dz \, dr \, d\theta$

   (c) Spherical. $\int_0^{\pi/4} \int_0^{2\pi} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

   (d) Cylindrical. $\int_0^{2\pi} \int_0^1 \int_{1-r^2}^{r^2-1} r \, dz \, dr \, d\theta$

(e) Rectangular. $\int_0^2 \int_0^{2-x} \int^{4-2x-2y} dz\,dy\,dx$

(f) Spherical. $\int_0^{2\pi} \int_0^{\pi} \int_0^{a} \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$

8. (a) $\int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \int_0^1 \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi = \pi/3$

(b) $\int_0^{\pi/2} \int_0^{2\pi} \int_0^{2\cos\phi} 2\rho^2 \sin\phi \; d\rho\, d\theta\, d\phi = 8\pi/4.$
This is twice the volume of sphere of radius 1 centered at $(0,0,1)$.

9. (a) Let $x/2 = u$ and $y/3 = v$, solve for u and v to get the transformation $T(u,v) = (2u, 3v)$. $\iint_S (2u+3v)6 \; du\, dv$ where S is the unit disk. Now use polar coordinates to get
$6\int_0^{2\pi} \int_0^1 (2r\cos\theta + 3r\sin\theta)\; r\, dr\, d\theta = 0$

(b) Let $T(u,v) = (u-v, 2u-v)$. Since $u = y - x$, we get $0 \le u \le 2$. Since $v = y - 2x$, we get $-1 \le v \le 0$. Compute $\frac{\partial(x,y)}{\partial(u,v)} = 1$.
$\int_0^2 \int_{-1}^0 u^2 \; dv\, du = 8/3$