Math 233 Exam 3 Solutions - Exam date 11/24/08

① Let $f(x) = xy + yz + zx$ then $\nabla f = (y+z,\ x+z,\ x+y)$

$\Rightarrow \vec{n} = \nabla f(2,0,3) = (3,5,2)$ so $(3,5,2)\cdot(x-2,\ y,\ z-3) = 0$

$\Rightarrow \underline{3x + 5y + 2z = 12}$

② a) $\nabla T = (-6x, -6y^2)$ so $\nabla T(1,-1) = (-6,-6)$

b) $\vec{u} = \dfrac{(3,-4)}{\|(3,-4)\|} = \left(\frac{3}{5}, -\frac{4}{5}\right)$, $D_u T(1,-1) = (-6,-6)\cdot\left(\frac{3}{5}, -\frac{4}{5}\right) = \underline{\underline{\dfrac{6}{5}}}$

c) Choose $\vec{x}$ such that $\vec{x}\cdot\nabla T(1,-1) = 0 \Rightarrow (x,y)\cdot(-6,-6) = 0 \Rightarrow -6x - 6y = 0$

$\Rightarrow x = -y$ so for example $(-1,1)$ is such a direction.

③ a) $f(1,-2) = z(1,-2) = 2$, $f_x(1,-2) = 1$, $f_y(1,-2) = -2$

b) $\nabla f(1,-2) = (f_x, f_y) = (1,-2)$

④ $V = xyz \Rightarrow dV = yz\,dx + xz\,dy + xy\,dz \Rightarrow \Delta V \approx dV = (100 + 200 + 200)(0.11)$

$= \underline{\underline{55}}$

(Note: actual $\Delta V = 55.4853\ldots$)

⑤ $\dfrac{\partial f}{\partial s} = \dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial s} + \dfrac{\partial f}{\partial z}\cdot\dfrac{\partial z}{\partial s} = (1)(6s) + (2yz)(3) + (y^2)(2s)$ where $\begin{cases} x = 8 \\ y = -2 \\ z = 0 \end{cases}$

$= 1\cdot 12 + 0 + 4\cdot 4 = \underline{\underline{28}}$

$\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial t} + \dfrac{\partial f}{\partial z}\cdot\dfrac{\partial z}{\partial t} = (1)(2) + (2yz)(-4t) + (y^2)(-2t) = 2 + 0 + 16 = \underline{\underline{18}}$

⑥ a) $\nabla f = (4x,\ 2y-4) \overset{set}{=} 0 \Rightarrow$ CP $(0,2)$

b) $g = x^2 + y^2$, $\nabla g = (2x, 2y) \Rightarrow (4x, 2y-4) = \lambda(2x, 2y)$ $\begin{cases} 4x = 2\lambda x \Rightarrow \lambda = 2 \text{ or } x = 0 \\ 2y - 4 = 2\lambda y \Rightarrow y - 2 = \lambda y \end{cases}$

$\lambda = 2 \Rightarrow y = -2$, $x^2 + y^2 = 9 \Rightarrow x = \pm\sqrt{5} \Rightarrow (\sqrt{5}, -2), (-\sqrt{5}, -2)$

$\Rightarrow (0,3), (0,-3)$

$x = 0 \Rightarrow y = \pm 3$

$f(\pm\sqrt{5}, -2) = 25$, $f(0,3) = 0$, $f(0,-3) = 24$, $f(0,2) = -1$

c) Max at $(\pm\sqrt{5}, -2)$, min at $(0,2)$

⑦ $\nabla f = (3x^2 - 3y,\ 3y^2 - 3x) \overset{set}{=} 0 \Rightarrow \begin{cases} x^2 = y \\ y^2 = x \end{cases} \Rightarrow y^4 - y = 0 = y(y^3 - 1) \Rightarrow y = 0, 1$

CP $(0,0)$, $(1,1)$

$\left.\begin{array}{ll} f_{xx} = 6x & f_{xy} = -3 \\ f_{yx} = -3 & f_{yy} = 6y \end{array}\right\} D = 36xy - 9 \Rightarrow$

$D(0,0) = -9 < 0$ so $\underline{(0,0) \text{ Saddle pt.}}$

$D(1,1) = 27 > 0$ and $f_{xx}(1,1) = 6 > 0$

so $\underline{(1,1) \text{ is minimum}}$.