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Business Calculus I (Math 221) Exam 3

Date: December 4, 2013 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Key

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Problem 1. Evaluate

(a) $\int \left(\frac{4}{t^2} + 3e^{9t} - \frac{2}{t}\right)\, dt$ $= \int 4t^{-2} + 3e^{9t} - 2t^{-1}\, dt$

$= -4t^{-1} + \frac{1}{3}e^{9t} - 2\ln|t| + C$

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(b) $\int \left(-5x^3 + 7\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)\, dx$ $= \int -5x^3 + 7x^{1/2} + x^{-1/3}\, dx$

$= -\frac{5}{4}x^4 + \frac{14}{3}x^{3/2} + \frac{3}{2}x^{2/3} + C$

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(c) $\int \frac{x^4}{\sqrt{3x^5+6}}\, dx$ $= \frac{1}{15}\int u^{-1/2}\, du = \frac{1}{15}\cdot 2u^{1/2} + C$

$u = 3x^5 + 6$

$du = 15x^4\, dx$

$= \frac{2}{15}\sqrt{3x^5+6} + C$

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Problem 2. If $\int f(x)\, dx = 4x^3 - 2x^{3/2} + 3e^x + C$, find $f(x)$.

$f(x) = \frac{d}{dx}\left(4x^3 - 2x^{3/2} + 3e^x\right) = 12x^2 - 3x^{1/2} + 3e^x$

125 + 10

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Problem 3. Evaluate

(a) $\int_{-10}^{10} x^5 \, dx$ $= \frac{1}{6} x^6 \Big|_{-10}^{10} = \frac{1}{6}(10^6 - 10^6) = 0$

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(b) $\int_{-2}^{2} (9x^2 - 4x + 3) \, dx$ $= \left[3x^3 - 2x^2 + 3x \right|_{-2}^{2}$

$= (3\cdot 8 - 2\cdot 4 + 3\cdot 2) - (3(-8) - 2\cdot 4 + 3(-2))$

$= 60$

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(c) $\int_{1}^{2} (2t-3)^{10} \, dt$ $= \frac{1}{2}\int_{-1}^{1} u^{10} du = \frac{1}{2}\cdot\frac{1}{11} u^{11} \Big|_{-1}^{1}$

$u = 2t-3$

$du = 2dt$ $= \frac{1}{22}(1-(-1))$

$x=1 \Rightarrow u=-1$

$x=2 \Rightarrow u=1$ $= \frac{1}{11}$

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(d) $\int_{0}^{1} x^2 e^{x^3} \, dx$ $= \frac{1}{3}\int_0^1 e^u du = \frac{1}{3} e^u \Big|_0^1$

$u = x^3$ $= \frac{1}{3}(e^1 - e^0)$

$du = 3x^2 dx$

$x=0 \Rightarrow u=0$ $= \frac{1}{3}(e-1)$

$x=1 \Rightarrow u=1$

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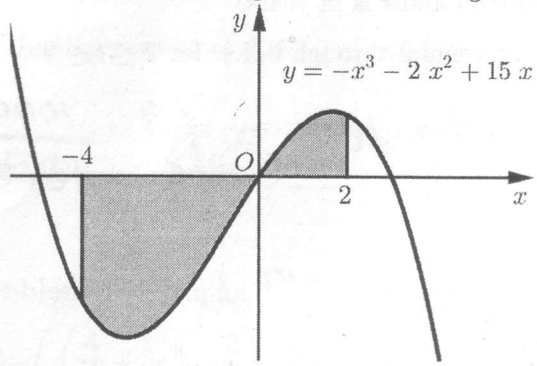
Problem 4. Express the shaded signed area under the given curve as an integral. Then evaluate the integral to find the signed shaded area under the curve.



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Signed area $= \int_{-4}^{2} -x^3 - 2x^2 + 15x \, dx = \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right]_{-4}^{2}$

$= \left(-\frac{1}{4}\cdot 16 - \frac{2}{3}\cdot 8 + \frac{15}{2}\cdot 4\right) - \left(-\frac{1}{4}\cdot 256 + \frac{2}{3}\cdot 64 + \frac{15}{2}\cdot 16\right) = -78$

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Bonus: Find the total (unsigned) shaded area bounded by the curve and the x–axis.

$A = -\int_{-4}^{0} -x^3 - 2x^2 + 15x \, dx + \int_{0}^{2} -x^3 - 2x^2 + 15x$

$A = \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right]_{0}^{-4} + \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right]_{0}^{2} = \frac{296}{3} + \frac{62}{3}$

$= \left(-\frac{1}{4}\cdot 256 + \frac{2}{3}\cdot 64 + \frac{15}{2}\cdot 16\right) + \left(-\frac{1}{4}\cdot 16 - \frac{2}{3}\cdot 8 + \frac{15}{2}\cdot 4\right) = 119\frac{1}{3}$

Problem 5. Find the total income over the next 5 years from a continuous income stream with annual flow rate $f(t) = 150\, e^{-0.2t}$.

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Income $= \int_{0}^{5} 150\, e^{-0.2t} \, dt$

$= \frac{150}{-0.2} e^{-0.2t} \Big|_{0}^{5}$

$= -750\left(e^{-1} - e^{0}\right)$

$= 750\left(1 - 1/e\right)$

$= \$474.09$

20+10

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Problem 6. The rate of increase in maintenance costs for a building is $M'(t) = \dfrac{1000}{\sqrt{t+7}}$, where M is in dollars and t is in years.
Find the total maintenance cost for years 2 through 9, i.e. for $2 \le t \le 9$.

$$\text{Total} = \int_2^9 M'(t)\,dt = \int_2^9 \frac{1000}{\sqrt{t+7}}\,dt$$

$u = t+7$
$du = dt$
$t=2 \Rightarrow u=9$
$t=9 \Rightarrow u=16$

$$= \int_9^{16} 1000\,u^{-1/2}\,du$$

$$= 1000 \cdot 2u^{1/2}\Big|_9^{16}$$

$$= 2000(\sqrt{16}-\sqrt{9}) = 2000$$

Problem 7. To produce x fenleys, the marginal cost in dollars is $\overline{MC} = 5x + 20$, and the marginal revenue is $\overline{MR} = 150 - 3x$. The fixed cost is \$2500.

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(a) Find the marginal profit function $\overline{MP}(x)$, where x is the number of fenleys.

$$\overline{MP} = \overline{MR} - \overline{MC} = (150-3x) - (5x+20)$$

$$= 130 - 8x$$

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(b) Find the profit function $P(x)$ for fenleys.

$$P(x) = \int \overline{MP}\,dx = \int 130 - 8x\,dx = 130x - 4x^2 + C$$

$$P(0) = R(0) - C(0) = -2500 \Rightarrow C = -2500$$

$$P(x) = 130x - 4x^2 - 2500$$

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(c) Find the profit when 100 fenleys are sold.

$$P(100) = (130)(100) - 4(100)^2 - 2500$$

$$= \$-29{,}500$$

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