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NAME: \_\_\_\_\_

Key

$$1 \text{ in} = 2.54 \text{ cm}, \quad 1 \text{ kg} = 2.2 \text{ lbs} \quad 1 \text{ gal} = 3.79 \ell$$

**Problem 1.** A right rectangular prism is 4 ft long, 2 ft wide, 3 ft deep.

(a) What is the volume of the prism?

$$24 \text{ ft}^3$$

(b) If a cube 6 in by 6 in by 6 in is filled with water, how many such cubes will fill the prism?

$$\begin{array}{l} \text{each} \\ \text{cube} \end{array} : \left(\frac{1}{2} \text{ ft.}\right)^3 = \frac{1}{8} \text{ ft}^3 / \text{cube} \quad \left| \quad \begin{array}{l} 24 \text{ ft}^3 \times \frac{8 \text{ cubes}}{1 \text{ ft}^3} \\ = 192 \text{ cubes} \end{array} \right.$$

**Problem 2.** (a) Explain the difference between an ounce and a fluid ounce.

Ounce = unit of weight (mass)

fl. oz = unit of volume

(b) How many ml are in 1 pt?

$$1 \text{ pt} \times \frac{1 \text{ gal}}{8 \text{ pts}} \times \frac{3.79 \ell}{\text{gal}} \times \frac{1000 \text{ ml}}{1 \ell} = 473 \text{ ml}$$

(c) Which is more water, a pound or a pint of water? Justify. use 1 ml = 1g water.

$$1 \text{ pt} \times \frac{473 \text{ ml}}{\text{pt.}} \times \frac{1 \text{ g}}{1 \text{ ml}} = 473 \text{ g} \quad \leftarrow \text{pint is more}$$

$$1 \text{ lb} \times \frac{1 \text{ kg}}{2.2 \text{ lbs}} \times \frac{1000 \text{ g}}{\text{kg}} = 454 \text{ g}$$

(d) Which is more, 1 oz or 1 fl oz of water? (1 lb = 16 oz, and 1 pt = 16 fl oz)

$$1 \text{ fl oz} = \frac{473}{16} \text{ g} \quad \leftarrow \text{fl oz is more} \quad 29.6 \text{ g}$$

$$1 \text{ oz} = \frac{454}{16} \text{ g} \quad 28.4 \text{ g}$$

5 pts each

Problem 3. (a) How many kilograms is one million centigrams?

$$10^6 \text{ cg} \times \frac{10^3 \text{ g}}{1000 \text{ cg}} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 10 \text{ kg}$$

(b) One acre is 43,560 sq ft. What is a square mile in acres?

$$1 \text{ sq mi} \times \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right)^2 \times \frac{1 \text{ acre}}{43560 \text{ ft}^2} = \frac{5280^2}{43560} \text{ acres} = 640 \text{ acres}$$

(c) The peregrine falcon is the fastest animal on earth, clocked diving at 80 m/sec. How fast is that in miles per hour?

$$80 \frac{\text{m}}{\text{sec}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 179 \text{ mph}$$

(d) The density of lead is 11 g/ml. What is that in pounds per gallon?

$$11 \text{ g/ml} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{2.2 \text{ lbs}}{\text{kg}} \times \frac{1000 \text{ ml}}{1 \text{ l}} \times \frac{3.79 \text{ l}}{\text{gal}} = 92 \text{ lbs/gal}$$

(e) The distance to Chicago is 800 miles. The distance to the moon is 380,000 km. How many trips to Chicago is one trip to the moon?

$$800 \text{ mi} \times \frac{1.6 \text{ km}}{\text{mi}} = 1280 \text{ km}$$

$$1 \text{ moon trip} \times \frac{380,000 \text{ km}}{1 \text{ moon trip}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{1 \text{ Chicago trip}}{800 \text{ mi}} = 295 \text{ trips to Chicago}$$

$$\frac{380,000}{1280} = 295$$

**Bonus** (f) A company produces 1500 toys per year. If one cup of glaze covers 5 toys, how many liters of glaze are needed each month?

$$1500 \frac{\text{toys}}{\text{yr}} \times \frac{1 \text{ cup}}{5 \text{ toys}} \times \frac{1 \text{ gal}}{16 \text{ cups}} \times \frac{3.79 \text{ l}}{\text{gal}} \times \frac{1 \text{ yr}}{12 \text{ months}} = 5.9 \text{ l/month}$$

**Bonus** Problem 4. The diameter of a human hair is about  $50 \mu\text{m}$ . ( $1 \mu\text{m}$  is called a micron.  $1 \text{ mm} = 1000$  microns.) If you laid hairs that are 1 in long next to each other, how many hairs would be needed to cover 1 square inch?

$$1 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{10,000 \mu\text{m}}{1 \text{ cm}} \times \frac{1 \text{ hair}}{50 \mu\text{m}} = 508 \text{ hairs}$$

5 pts each

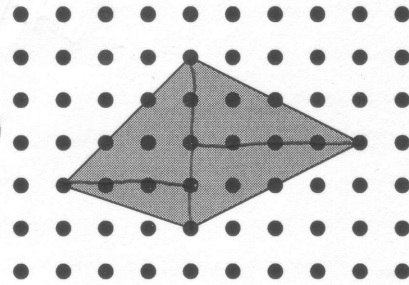
25 + 10

**Problem 5.** (a) If the dots are spaced 1 cm apart, compute the area of the quadrilateral shown.

4 triangles area:

$$\frac{1}{2}(3 \cdot 3) + \frac{1}{2}(2 \cdot 4) + \frac{1}{2}(2 \cdot 4) + \frac{1}{2}(1 \cdot 3)$$

$$= 14 \text{ cm}^2$$

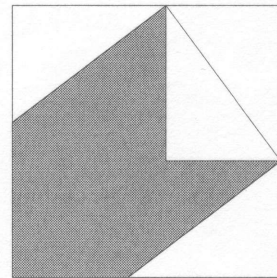


(b) What is the area of a quadrilateral with perpendicular diagonals that are 8 cm and 10 cm long?

$$\frac{1}{2}(8 \cdot 10) = 40 \text{ cm}^2$$

**Problem 6.** Recall our proof of the Pythagorean Theorem. Given four copies of a right triangle with sides  $a, b, c$  such that  $a^2 + b^2 = c^2$ , what is the area of the shaded region?

$c^2$



**Problem 7.** (a) A kite is flying on a 25 m line, and its shadow is 7 m away. If the sun is directly overhead, how high is the kite? Justify.

$$\sqrt{25^2 - 7^2} = 24 \text{ m}$$

(b) If a triangle has side lengths 8, 15, 18, can it be a right triangle? Justify.

NO.  $8^2 + 15^2 \neq 18^2$

(c) What is the longest pole that can fit in a box that is 4 feet long, 3 feet wide, and 5 feet tall? Show work.

$$\sqrt{4^2 + 3^2 + 5^2} = \sqrt{50} = 5\sqrt{2} \approx 7.07 \text{ ft.}$$

5 pts each

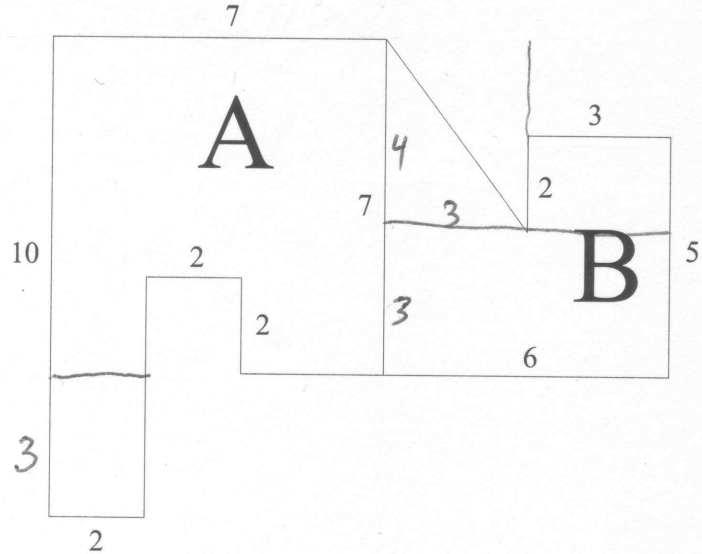
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**Problem 8.** (a) Compute the area of region A. (b) Compute area of region B.  
Show work.

6 pts. a.)  $49 + 6 - 4$   
 $= \text{51}$

6 pts. b.)  $18 + 6 + \frac{1}{2} \cdot 12$   
 $= 30$

12 pts. OR  $42 - 6 - \frac{1}{2} \cdot 12$   
 $= 30$



(15 pts.) **Problem 9. (BONUS)** Sam reports that a plate weighs 2 lbs. Alice reports that a vase weighs 2.0 lbs. Bob can weigh things in pounds to two decimal places.

(a) If Alice weighs the plate, what weight range could she report?

$1.5 - 2.4$  lbs

(b) If Bob weighs the vase, what weight range could he report?

$1.95 - 2.04$  lbs

(c) If Bob weighs the plate, what weight range could he report?

$1.50 - 2.49$  lbs