NAME: Key

Math 214 - Exam 2

Justify answers and show all work for full credit.

Problem 1:
A certain gas tank is designed to hold 15 gallons. Suppose that the volumes of randomly selected gas tanks are approximately normal with mean 15.0 gallons and standard deviation 0.15 gallons.

(a) The manufacturer will exclude the largest 2% of gas tanks. How large does a gas tank have to be for it to be excluded?

(b) What proportion of gas tanks will hold between 14.75 and 15.10 gallons?

(c) If a sample of 16 gas tanks is randomly selected, what is the probability that the sample mean will be between 14.75 and 15.10 gallons?

4 a) $z = 2.055$ for $P = 2\%$ from table A

$15 + (2.055)(0.15) = \underline{15.31 \text{ gallons}}$

6 b) $z = \dfrac{14.75 - 15}{0.15} = -1.67$ $\quad P = 0.0475$

$z = \dfrac{15.1 - 15}{0.15} = 0.67$ $\quad$ ~~Exclusion~~ $\underline{P = 0.7486}$

$P(-1.67 \le z \le 0.67) = 0.7486 - 0.0475$

$= 0.7011 = \underline{70.11\%}$

6 c) $\mu_{\bar{x}} = 15.0$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = \dfrac{0.15}{\sqrt{16}} = 0.0375$

$z = \dfrac{14.75 - 15}{0.0375} = -6.67$ $\quad P(-6.67 \le z \le 2.67)$

$= 0.9962 - 0.00$

$z = \dfrac{15.1 - 15}{0.0375} = 2.67$ $\quad \approx 99.6\%$

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Problem 2:

A drug is found to be 90% effective in curing a certain disease.

(a) If 500 people are treated with the drug, what is the expected number of patients who will be cured?

(b) What is the standard deviation of the number of patients cured in a sample of size 100?

(c) If 100 people are given the drug, what is the probability that exactly 99 will be cured?

(d) If 500 are treated, find the probability that more than 400 will be cured. (Use a normal approximation.)

2 a) $\mu = (0.9)(500) = 450$

3 b) $\sigma = \sqrt{np(1-p)} = \sqrt{100(0.9)(0.1)} = 3$

5 c) $P(X = 99) = \binom{100}{99}(0.9)^{99}(0.1)^{1} = 0.000295$

$\approx 0.03\%$

6 d) $P(X > 400)$ using $N(450, 6.71)$

$\mu = 450$ (part a)

$\sigma = \sqrt{500(0.9)(0.1)} = 6.71$

$z = \dfrac{400 - 450}{6.71} = -7.45$

$P(z \geq -7.45) \approx 100\%$

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Problem 3:
Scores on an IQ test are normally distributed with standard deviation $\sigma = 10$. In a simple random sample of 36 people, the mean score is 104.

(a) Based on this data, what is the 95% confidence interval for the population mean IQ score?

(b) How many people should be tested to reduce the margin of error in half?

a) $m = z^* \frac{\sigma}{\sqrt{n}} = (1.96)\left(\frac{10}{\sqrt{36}}\right) = 3.27$

6 95% CI = $104 \pm 3.27 = (100.73, 107.27)$

4 b) $4n = 144$ people

Problem 4:
A statistician in a new city tests a realtor's claim that the average rent μ is more than \$650. Based on available data, he finds that a 95% confidence interval for μ is (\$630, \$674).

(a) State the hypotheses H_0 and H_A

2 H_0 : $\mu = 650$ H_A : $\mu > 650$

(b) Would he reject or fail to reject H_0 at the 0.05 confidence level? Why?

4 Based on 95% CI, $P(\cancel{z} > 0) \simeq 50\%$, much more than $\alpha = 0.05$

So fail to reject H_0

[To make correct, should have been 90% CI]
Since 1-sided test

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Problem 5:

On a certain snow-covered track, the mean stopping distance is 215m with standard deviation $\sigma = 2.5m$. A tire company claims that its new snow tires can perform better. A random sample of 9 snow tires had a mean stopping distance of 213m. Is the improvement in the new snow tires statistically significant at the 0.05 confidence level?

(a) State the hypotheses H_0 and H_A

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H_0 : $\mu = 215$ H_A : $\mu < 215$

(b) Specify your test statistic and its sampling distribution. For the t-distribution, specify its degrees of freedom.

(c) Compute the test statistic.

(d) Estimate or compute the P-value as accurately as possible using the tables (or your calculator).

(e) Would you Reject or Fail to reject (accept) H_0 at the given significance level? What does that mean in this case?

2 b) z-statistic, normal distribution

4 c) $z = \frac{213-215}{2.5}\sqrt{9} = -2.4$

4 d) $P(z \leq -2.4) = 0.0082$

4 e) Reject H_0. Yes, tires are significantly better.

[Critical $z^* = -1.645$]

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Problem 6:
A double-blind, randomized, 24-month trial compared a simvastatin group and a combined-therapy group. The main issue of the study was the change in thickness of arterial walls. Suppose the data, in some units, are given by the following summaries:

	n	xbar	s
combined therapy	20	11.1	5.04
simvastatin	15	5.8	5.00

Perform a two-sided significance test of equality of population means using a 0.05 significance level.

(a) State the hypotheses H_0 and H_A

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H_0 : $\mu_1 = \mu_2$ H_A : $\mu_1 \neq \mu_2$

(b) Specify your test statistic and its sampling distribution. For the t-distribution, specify its degrees of freedom.

(c) Compute the test statistic.

(d) Estimate or compute the P-value as accurately as possible using the tables (or your calculator).

(e) For given significance level, indicate if the difference is statistically significant.

2 b) t-statistic, t-distribution, $df = 14$

4 c) $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \dfrac{5.3}{1.71} = 3.10$

ie. $2P \simeq 0.006 < \alpha = 0.05$

4 d) $P(T \geq 3.10) \simeq 0.003$, much less than $\frac{\alpha}{2} = 0.025$

4 e) Reject H_0, yes difference is statistically significant

[Critical $t^* = 2.145$]

$5.3 \pm (2.145)(1.71)$

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Problem 7:
A machine is supposed to fill jugs with 120 ounces of detergent. Below is a summary of a random sample of quality control measurements:

n	xbar	s
25	126	4.2

$df = 24$ for t-distr.

(a) Find the 90% confidence interval for the mean amount of detergent.

(b) Is the machine working to specifications at this confidence level?

a) $m = t^* \frac{s}{\sqrt{n}} = (1.711)\left(\frac{4.2}{5}\right) = 1.44$

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90% CI : $126 \pm 1.44 = (124.56, 127.44)$

2 b) No, 120 is outside 90% CI.

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Problem 8:

In 2006 the EPA revised how it computes MPG ratings on new cars. The old test favored hybrid vehicles, as the test conditions were optimal for a hybrid's use of electric power at low speeds. Under the old standard the estimated MPG for a Toyota Prius was 60 MPG. Under the new test, with more real-world driving, the Prius was estimated to get 48 MPG in the city.

Ten Prius owners decide to test whether the new test is accurate, and record their MPG data:

Car:	1	2	3	4	5	6	7	8	9	10	n	xbar	s
MPG:	51	50	46	51	51	58	48	55	52	53	10	51.5	3.37

Perform a two sided significance test with null hypothesis that the new standards are accurate. Use $\alpha = .05$.

(a) State the hypotheses H_0 and H_A

2 H_0 : $\mu = 48$ H_A : $\mu \neq 48$

(b) Specify your test statistic and its sampling distribution. For the t-distribution, specify its degrees of freedom.

(c) Compute the test statistic.

(d) Estimate or compute the P-value as accurately as possible using the tables (or your calculator).

(e) Would you Reject or Fail to reject (accept) H_0 at the given significance level? What does that mean in this case?

2 b) t-statistic, t-distr. $df = 9$

4 c) $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{51.5 - 48}{3.37/\sqrt{10}} = 3.28$

4 d) $P(T \geq 3.28) \approx 0.005$

4 e) $2P = 0.01 < \alpha = 0.05$, so Reject H_0

No, new standards do not seem to be accurate

[Critical $t^* = 2.262$]

$51.5 \pm (2.262)(1.065)$

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