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Business Calculus I (Math 221) Exam 3

Date: November 23, 2016 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Key

Problem 1. Suppose x and y satisfy $4\sqrt{x} + xy^3 - 2 = x^2 - 6y$. Find $\frac{dy}{dx}$ at the point $(4, 1)$.

4 $$\frac{2}{\sqrt{x}} + 3xy^2\frac{dy}{dx} + y^3 = 2x - 6\frac{dy}{dx}$$

6 $$(4,1) \Rightarrow \frac{2}{2} + 12\frac{dy}{dx} + 1 = 8 - 6\frac{dy}{dx} \Rightarrow 18\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{1}{3}$$

Problem 2. Find the derivatives $\frac{dy}{dx}$.

(a) $y = \ln(7x^3 - 9x - 3)$

4 $$\frac{dy}{dx} = \frac{21x^2 - 9}{7x^3 - 9x - 3}$$

(b) $y = e^{(-4x+3)} + \frac{9}{x} - 3$

5 $$\frac{dy}{dx} = -4e^{-4x+3} - \frac{9}{x^2}$$

(c) $e^{6y} + \ln(y) = \frac{5}{\sqrt{x}} + x$

6 $$6e^{6y}\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = -\frac{5}{2}x^{-3/2} + 1$$

3 $$\frac{dy}{dx} = \frac{(-5/2)x^{-3/2} + 1}{6e^{6y} + \frac{1}{y}}$$

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Problem 3. Evaluate

(a) $\int 3x^4 + \frac{2}{x^6} + \frac{4}{x} - 10\ dx$

8 $\int 3x^4 + 2x^{-6} + 4x^{-1} - 10\ dx = \frac{3}{5}x^5 - \frac{2}{5}x^{-5} + 4\ln|x| - 10x + C$

(b) $\int 4x^{3/5} - 3e^{7x} - \sqrt[3]{x} + \frac{13}{x^4}\ dx$

8 $= \frac{5}{2}x^{8/5} - \frac{3}{7}e^{7x} - \frac{3}{4}x^{4/3} + \frac{13}{3}x^{-3} + C$

(c) $\int x^4\sqrt{2x^5+3}\ dx$ $\quad u = 2x^5+3$, $du = 10x^4\,dx$

6 $\frac{1}{10}\int \sqrt{u}\,du = \frac{1}{15}(2x^5+3)^{3/2} + C$

$\frac{1}{10}\cdot\frac{2}{3}\cdot u^{3/2} + C$

(d) $\int \frac{3x^8}{x^9-2}\ dx$ $\quad u = x^9-2$, $du = 9x^8\,dx$

$= \frac{3}{9}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

6 $= \frac{1}{3}\ln|x^9-2| + C$

Problem 4. Some kids rolling a snowball to make a snowman can add $80\,\text{cm}^3/\text{min}$ of snow to the ball. How fast is the radius of the snowball increasing when the radius is 10 cm? (For a sphere, $V = \frac{4}{3}\pi r^3$.)

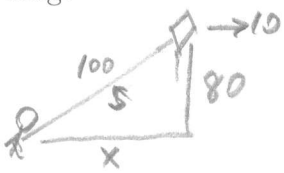
$$V = \frac{4}{3}\pi r^3$$

2 $$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$80 = 4\pi (10)^2 \frac{dr}{dt}$$

6 $$\frac{dr}{dt} = \frac{80}{400\pi} = \frac{1}{5\pi} \text{ cm/min}$$

Problem 5. The wind blows a kite 80 ft above the ground at 10 ft/sec parallel to the ground. How fast must the string be let out when the string is 100 ft long?



4 $$x^2 + 80^2 = s^2$$

2 $$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt}$$

$$s = 100 \Rightarrow x = 60 \quad (60^2 + 80^2 = 100^2)$$

6 $$2(60)(10) = 2(100)\frac{ds}{dt}$$

$$\Rightarrow \frac{ds}{dt} = 6 \text{ ft/sec}$$

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Problem 6. Suppose a bond that now costs \$2,000 will return \$5,000 in 30 years.

(a) If compounded continuously, what is the interest per year (APR) on the bond?

(b) How much will the bond be worth after 10 years?

a) $P(t) = Pe^{rt}$

$5000 = 2000\, e^{(r)(30)} \Rightarrow \frac{5}{2} = e^{30r}$

6 $r = \frac{\ln(5/2)}{30} \simeq 0.03054 \simeq \underline{3.054\%}$

4 b) $P(10) = 2000\, e^{0.3054} \simeq \underline{\$2714.42}$

Problem 7. To produce x flerds, the marginal cost in dollars is $\overline{MC} = 4x + 30$, and the marginal revenue is $\overline{MR} = 110$. The fixed cost for making flerds is \$1,000.

(a) Find the marginal profit function $\overline{MP}(x)$, where x is the number of flerds.

(b) Find the profit function $P(x)$ for flerds.

(c) After how many flerds, if ever, will selling flerds be profitable? Explain.

2 a) $MP = MR - MC = 110 - (4x+30) = \underline{80 - 4x}$

b) $P = \int MP = \int 80 - 4x\, dx = 80x - 2x^2 + C$

Fixed Cost \$1000 $\Rightarrow$ $\underline{P(x) = 80x - 2x^2 - 1000}$

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c) $MP = 0 \Rightarrow 80 - 4x = 0 \Rightarrow x = 20$

6 $P(20) = \$200$

This is a max since $P''(x) = -4 < 0$ (CD)

2 So never profitable to sell flerds.