MTH 231 Exam 3 Solutions 4/10/2006

① $h(x) = x^3 - 6x^2 + 15 \quad -5 \le x \le 5$

$h'(x) = 3x^2 - 12x = 3x(x-4) \overset{set}{=} 0 \Rightarrow$ CP: $x = 0, 4$

Test Points $\underbrace{h(-5) = -260}_{min}$, $\underbrace{h(0) = 15}_{max}$, $h(4) = -17$, $h(5) = -10$

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② $f(x) = x^4 - 2x^2 - 3$

$f'(x) = 4x^3 - 4x = 4x(x-1)(x+1) \overset{set}{=} 0$

a) $\Rightarrow$ CP: $x = -1, 0, 1$

		-1		0		1	
$4x$	$-$		$-$		$+$		$+$
$x-1$	$-$		$-$		$-$		$+$
$x+1$	$-$		$+$		$+$		$+$
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	dec		inc		dec		inc

By 1st Deriv Test rel. min at $x = -1, 1$ and rel. max at $x = 0$

$f(x)$ decreasing on $(-\infty, -1) \cup (0, 1)$, increasing on $(-1, 0) \cup (1, \infty)$

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b) $g(x) = x^2 - 8x + 6\ln x, \quad x > 0$

$g'(x) = 2x - 8 + 6/x \overset{set}{=} 0 \Leftrightarrow 2x^2 - 8x + 6 = 0 \quad (x > 0)$

$(x-3)(x-1) = 0$ CP: $x = 1, 3$

		1		3	
$x-1$	$-$		$+$		$+$
$x-3$	$-$		$-$		$+$
$g'(x)$	$+$		$-$		$+$
$g(x)$	inc		dec		inc

By 1st Deriv Test, rel. max at $x = 1$, rel. min at $x = 3$

$g(x)$ increasing on $(-\infty, 1) \cup (3, \infty)$, decreasing on $(1, 3)$.

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③ $f(x) = x^4 - 2x^2 - 3$

$f'(x) = 4x^3 - 4x = 4x(x-1)(x+1) \overset{set}{=} 0 \Rightarrow$ CP: $x = -1, 0, 1$

$f''(x) = 12x^2 - 4 = 4(3x^2 - 1) \overset{set}{=} 0 \quad x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

a) By 2nd deriv test: $f''(-1) > 0 \Rightarrow x = -1$ rel. min

$f''(0) < 0 \Rightarrow x = 0$ rel max

$f''(1) > 0 \Rightarrow x = 1$ rel. min

		$-\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$	
$x - \frac{1}{\sqrt{3}}$	$-$		$-$		$+$
$x + \frac{1}{\sqrt{3}}$	$-$		$+$		$+$

PI: $x = -\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$

$f''(x)$	$+$	$-$	$+$
f	CU	CD	CU
	$(-\infty, -\frac{1}{\sqrt{3}})$	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}}, \infty)$

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b) $g(x) = x^2 - 8x + 6\ln x$ CP: $x = 1, 3$ by (2b).

$g'(x) = 2x - 8 + 6/x$

$g''(x) = 2 - 6/x^2 \overset{set}{=} 0, \quad 2x^2 = 6 \quad (x > 0) \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

		$-\sqrt{3}$		$\sqrt{3}$	
$2 - 6/x^2$	$+$		$-$		$+$
$g(x)$	CU		CD		CU

Domain $x > 0 \Rightarrow g(x)$ CD on $(0, \sqrt{3})$, CU on $(\sqrt{3}, \infty)$.

By 2nd Deriv Test, $g''(1) < 0 \Rightarrow$ rel. max, $g''(3) > 0$ rel. min

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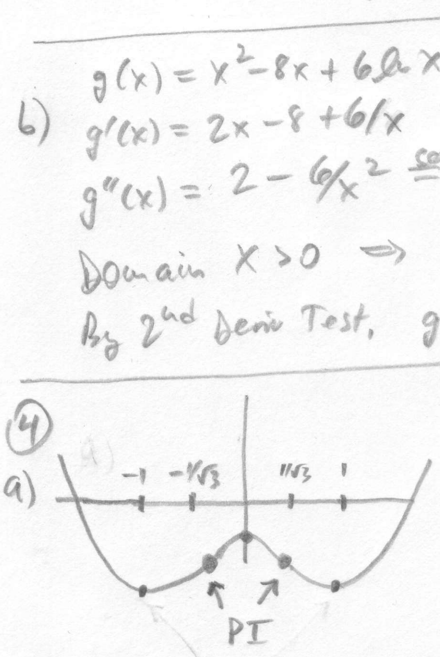
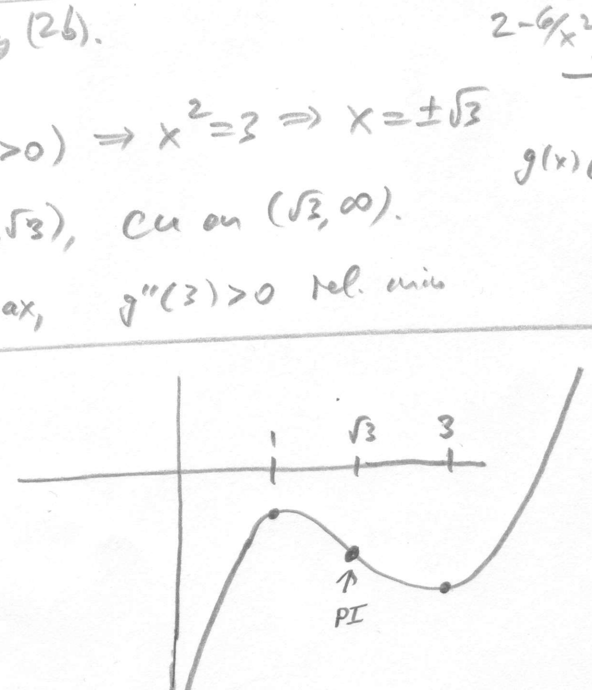
④

a)



b)



⑤ $V = \pi r^2 h = 4000 \text{ cm}^3 \Rightarrow h = \dfrac{4000}{\pi r^2}$

$C = 2 \cdot 2\pi r^2 + 1 \cdot 2\pi r h$

$C(r) = 4\pi r^2 + 2\pi r\left(\dfrac{4000}{\pi r^2}\right) = 4\pi r^2 + \dfrac{8000}{r}$

$C'(r) = 8\pi r - \dfrac{8000}{r^2} \overset{\text{set}}{=} 0 \Rightarrow 8\pi r^3 = 8000$

$r^3 = \dfrac{1000}{\pi}$

Note: $C''(r) = 8\pi + \dfrac{16000}{r^3} > 0$

$r = \dfrac{10}{\sqrt[3]{\pi}}, \quad h = \dfrac{4000}{\pi \cdot 100} \cdot \pi^{2/3}$

so this is a rel. min by 2nd deriv. test.

$h = 40/\sqrt[3]{\pi}$

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⑥ $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

$e^{x+\Delta x} \approx e^x + e^x \Delta x$

$e^{0.9} = e^{1-0.1} \approx e' + e'(-0.1) = e - \dfrac{e}{10} = \dfrac{9e}{10}$

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⑦ $x_{n+1} = x_n - \dfrac{x^4 - x - 1}{4x^3 - 1}$

$x_1 = 1$ 1.0

$x_2 = 1 - \dfrac{-1}{3} = \dfrac{4}{3}$ $\underline{\underline{1.33}}$

$x_3 = \dfrac{4}{3} - 0.0975\overline{3}$ $\underline{\underline{1.2358}}$

Ans. $\underline{\underline{1.2}}$

$x_4 = 1.2358 - 0.0148$ $\underline{\underline{1.2210}}$