Problem 1 (32pts). Compute the derivative $\frac{dy}{dx}$. Do not simplify. Show all work!

(a) $y = \frac{e^{5x}}{7 + \cos(3x)}$

$$y' = \frac{(7 + \cos(3x))(5e^{5x}) - (e^{5x})(-3\sin(3x))}{(7 + \cos(3x))^2}$$

(b) $y = (\sqrt{7x} + \sqrt{x^2 + 4})^{14}$

$$y' = 14(\sqrt{7x} + \sqrt{x^2 + 4})^{13}(\frac{7}{3}(7x)^{-\frac{1}{2}} + \frac{1}{2}(x^2 + 4)^{-\frac{1}{2}}(2x))$$

(c) $y = \ln(2 + \tan(3x + 4))$

$$y' = \frac{3\sec^2(3x+4)}{2 + \tan(3x+4)}$$

(d) $xe^y = y - 1$

$$xe^y \frac{dy}{dx} + e^y = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{e^y}{1-xe^y}$$
Problem 2 (20pts). Let \( f(x) = \frac{1}{2x + 3} \).

(a) Use the definition of the derivative to find \( f'(1) \).

\[
5 \quad f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{1}{2(1+h) + 3} - \frac{1}{5} = \lim_{h \to 0} \frac{1}{h} \left( \frac{1}{5+2h} - \frac{1}{5} \right)
\]

\[
5 \quad = \lim_{h \to 0} \frac{1}{h} \left( \frac{5 - (5+2h)}{5(5+2h)} \right) = \lim_{h \to 0} \frac{1}{h} \left( \frac{-2h}{25+10h} \right) = -\frac{2}{25}
\]

(b) Use any method to find \( f''(1) \).

\[
2 \quad f''(1) = \frac{2}{(2+3)^2} = -2 \left( \frac{2}{x+3} \right)^2
\]

\[
5 \quad f''(1) = 4 \left( \frac{2}{x+3} \right)^{-3} (2) = \frac{8}{(2+3)^3}
\]

\[
3 \quad f''(1) = \frac{8}{5^3} = \frac{8}{125}
\]

Problem 3 (18pts). Graphs of \( f(x) \) and \( g(x) \) are shown above. Show all work below!

(a) Let \( A(x) = f(x)g(x) \). Find \( A'(2) \).

\[
A'(x) = f(x)g'(x) + f'(x)g(x)
\]

\[
2 \quad A'(2) = f(2)g'(2) + f'(2)g(2)
\]

\[
4 \quad = (1) \cdot (2) + (-1) \cdot (4) = -2
\]

(b) Let \( B(x) = f(g(x)) \). Find \( B'(2) \).

\[
B'(x) = f'(g(x)) \cdot g'(x)
\]

\[
2 \quad B'(2) = f'(g(2)) \cdot g'(2)
\]

\[
4 \quad = f'(4) \cdot (2) = (3)(2) = 6
\]

(c) Let \( C(x) \) be the inverse of \( g(x) \) for \( 0 \leq x \leq 3 \). Find \( C(2) \) and \( C'(2) \).

\[
2 \quad g(1) = 2 \Rightarrow C(2) = 1
\]

\[
4 \quad C'(2) = \frac{1}{g'(1)} = \frac{1}{2}
\]
Problem 4 (10pts). Find the two points on the ellipse \(x^2 + 2y^2 = 1\) where the tangent line has slope 1.

\[
\begin{align*}
2x + 4y \frac{dy}{dx} &= 0 \\
\Rightarrow \quad 2x + 4y (1) &= 0 \\
\Rightarrow \quad x &= -2y
\end{align*}
\]

\[
\begin{align*}
(2y)^2 + 2y^2 &= 1 \\
\Rightarrow \quad 4y^2 + 2y^2 &= 1 \\
\Rightarrow \quad y^2 &= \frac{1}{4}
\end{align*}
\]

\[
\begin{align*}
y &= \pm \frac{1}{\sqrt{4}} \\
x &= -2y \\
\Rightarrow \quad \left(-\frac{2}{\sqrt{4}}, \frac{1}{\sqrt{4}}\right) \quad \text{and} \quad \left(\frac{2}{\sqrt{4}}, -\frac{1}{\sqrt{4}}\right)
\end{align*}
\]

Problem 5 (12pts). A ball is thrown up from the ground with a velocity of 80 ft/sec.

(a) Find the maximum height of the ball.

\[
S(t) = 80t - 16t^2
\]

\[
\begin{align*}
V(t) &= S'(t) = 80 - 32t = 0 \\
\Rightarrow \quad t &= \frac{80}{32} = \frac{5}{2} = 2.5\ \text{sec}
\end{align*}
\]

\[
S(2.5) = (80)(\frac{5}{2}) - 16(\frac{25}{4})
\]

\[
= 200 - 100 = 100\ \text{ft}.
\]

(b) Find the velocity of the ball when it is 96 feet above the ground (on its way up).

\[
S(t) = 80t - 16t^2 = 96 \Rightarrow 16t^2 - 80t + 96 = 0
\]

\[
(t-2)(t-3) = 0 \Rightarrow t = 2 \quad \text{or} \quad t = 3\ \text{sec}
\]

\[
V(2) = 80 - 32(2) = 80 - 64 = 16\ \text{ft/sec}
\]

Problem 6 (15pts). A baseball diamond is a square with sides 30 yards. A batter hits the ball and runs toward first base with a speed of 8 yd/sec. At what rate is his distance to second base decreasing when he is halfway to first base?

\[
x^2 + 30^2 = y^2
\]

\[
2x \frac{dx}{dt} = 2y \frac{dy}{dt}
\]

\[
\frac{dy}{dt} = -8 \quad \text{when} \quad x = 15
\]

\[
\begin{align*}
(2)(18)(-8) &= (2)(18\sqrt{5}) \frac{dy}{dt} \\
\Rightarrow \quad \frac{dy}{dt} &= -8\sqrt{5} = -3.6\ \text{yd/sec}
\end{align*}
\]