Calculus I (Math 231) Exam 2

Date: November 5, 2007 — Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Key

Problem 1. *Compute the derivative of the following functions. Do not simplify. Show all work!*

5 (a) $f(x) = \dfrac{3x-2}{\sqrt{2x+1}}$

$$\frac{\left(\sqrt{2x+1}\right)(3) - (3x-2)\left(\frac{1}{2}(2x+1)^{-1/2}\cdot 2\right)}{(2x+1)} = \frac{3x+5}{(2x+1)^{3/2}}$$

5 (b) $f(x) = \cos^2(x^3)$

$$2\cos(x^3)\cdot(-\sin(x^3))\cdot 3x^2$$

missing term 3/5

5 (c) $f(x) = \sqrt[3]{x}\, e^{-(x^2+2)}$

$$\sqrt[3]{x}\, e^{-(x^2+2)}\cdot(-2x) + e^{-(x^2+2)}\left(\frac{1}{3}x^{-2/3}\right)$$

10 **Problem 2.** *Suppose x and y satisfy* $2x + x^2y^2 + \sin(3y) = 2$. *Find $\dfrac{dy}{dx}$ at the point $(1,0)$.*

(6) $$2 + x^2\left(2y\frac{dy}{dx}\right) + 2xy^2 + \cos(3y)\cdot 3\frac{dy}{dx} = 0$$

(4) $$2 + 3\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2}{3}$$

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Problem 3. *Let* $f(x) = \sqrt{3+5x}$.

10 (a) *Use the definition of the derivative to find* $f'(1)$. 5

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{3+5(1+h)} - \sqrt{8}}{h} \cdot \frac{\sqrt{8+5h}+\sqrt{8}}{\sqrt{8+5h}+\sqrt{8}}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}(\sqrt{8+5h}+\sqrt{8})} = \frac{5}{2\sqrt{8}}$$

10 (b) *Use any method to find* $f''(1)$.

$$f'(x) = \frac{1}{2}(3+5x)^{-1/2} \cdot 5$$

$$f''(x) = \frac{5}{2} \cdot \left(-\frac{1}{2}\right)(3+5x)^{-3/2} \cdot 5$$

$$f''(1) = \frac{5}{2} \cdot \left(-\frac{1}{2}\right) \cdot 8^{-3/2} \cdot 5 = -\frac{25}{4} \cdot 8^{-3/2}$$

10 **Problem 4.** *A bullet is fired up from the ground with initial velocity of 3200 ft/sec.*

(a) *Find the maximum height of the bullet.*

$$s(t) = -16t^2 + 3200t$$

$$s'(t) = -32t + 3200 \overset{set}{=} 0 \Rightarrow t = 100$$

$$s(100) = -160000 + 320000 = 160{,}000 \text{ ft.}$$

(b) *Find the velocity of the bullet when it returns to the ground.*

$$s(t) \overset{set}{=} 0 \Rightarrow 16t^2 = 3200t$$

$$t^2 = 200t \Rightarrow t = 200$$

$$s'(200) = (-32)(200) + 3200 = -3200 \text{ ft/sec}$$

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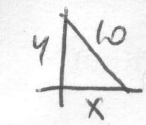
15 **Problem 5.** *A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?*



$$x^2+y^2=100$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

If $x=6$, $y=8$; also $\frac{dx}{dt}=1$

$$2\cdot 6\cdot 1+2\cdot 8\frac{dy}{dt}=0 \Rightarrow \frac{dy}{dt}=-\frac{12}{16}=-\frac{3}{4}$$

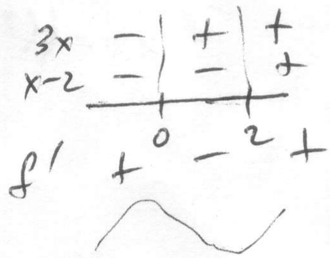
Problem 6. *Let* $f(x)=x^3-3x^2+1$.

5 (a) *Find the critical points.*

5 (b) *Find intervals where $f(x)$ is increasing or decreasing.*

5 (c) *Identify all relative extrema using the First Derivative Test.*

5 (d) *Identify the absolute max and min of $f(x)$ for $1\le x\le 3$.*



a) $f'(x)=3x^2-6x=3x(x-2)$

critical pts $x=0$, $x=2$

b) Incr $x>2$, $0>x$

Decr. $0<x<2$

c) 0 is max, 2 is min

d) $f(1)=1-3+1=-1$

$f(2)=8-12+1=-3$ min $x=2$

$f(3)=27-27+1=1$ max $x=3$

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