Problem 1. Compute the derivative of the following functions. Do not simplify. Show all work!

5. (a) \[ f(x) = \frac{3x - 2}{\sqrt{2x + 1}} \]
   \[ = \frac{(\sqrt{2x + 1})(3) - (3x - 2)\left(\frac{1}{2}(2x + 1)^{-\frac{1}{2}}\right)}{(2x + 1)} \]

5. (b) \[ f(x) = \cos^2(x^3) \]
   \[ = 2\cos(x^3)(\frac{1}{2}\sin(x^3)) \cdot 3x^2 \]

5. (c) \[ f(x) = \sqrt{x}e^{-(x^2 + 2)} \]
   \[ = \frac{3}{\sqrt{x}}e^{-(x^2 + 2)} \cdot (-2x) + e^{-(x^2 + 2)} \left(\frac{1}{3}x^{-2/3}\right) \]

Problem 2. Suppose \( x \) and \( y \) satisfy \( 2x + x^2y^2 + \sin(3y) = 2 \).
Find \( \frac{dy}{dx} \) at the point \( (1, 0) \).

(6) \[ 2 + x^2(2y \frac{dy}{dx}) + 2xy^2 + \cos(3y) - 3 \frac{dy}{dx} = 0 \]

(4) \[ 2 + 3 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2}{3} \]
Problem 3. Let \( f(x) = \sqrt{3 + 5x} \).

(a) Use the definition of the derivative to find \( f'(1) \).

\[
\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\sqrt{3+5(1+h)} - \sqrt{3+5}}{h} = \lim_{h \to 0} \frac{5}{\sqrt{3+5h} + \sqrt{3} + 2h} = \frac{5}{2\sqrt{8}}
\]

(b) Use any method to find \( f''(1) \).

\[
f'(x) = \frac{5}{2} (3+5x)^{-1/2} \cdot 5 = \frac{25}{2} (3+5x)^{-3/2}
\]

\[
f''(1) = \frac{25}{4} \cdot 8^{-3/2} = \frac{25}{4} \cdot 8^{-3/2}
\]

Problem 4. A bullet is fired up from the ground with initial velocity of 3200 ft/sec.

(a) Find the maximum height of the bullet.

\[s(t) = -16t^2 + 3200t\]

\[s'(t) = -32t + 3200 \quad \text{set} \quad 0 \quad \Rightarrow \quad t = 100\]

\[s(100) = -160000 + 320000 = 160000 \text{ ft}.
\]

(b) Find the velocity of the bullet when it returns to the ground.

\[s(t) = 0 \quad \Rightarrow \quad 16t^2 = 3200t\]

\[t^2 = 200t \quad \Rightarrow \quad t = 200\]

\[s'(200) = (-32)(200) + 3200 = -3200 \text{ ft/sec}\]
Problem 5. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

\[ y = 10 \quad x^2 + y^2 = 100 \]

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]

If \( x = 6 \), \( y = 8 \); also \( \frac{dx}{dt} = 1 \)

\[ 2 \cdot 6 \cdot 1 + 2 \cdot 8 \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{12}{16} = -\frac{3}{4} \]

Problem 6. Let \( f(x) = x^3 - 3x^2 + 1 \).

(a) Find the critical points.

(b) Find intervals where \( f(x) \) is increasing or decreasing.

(c) Identify all relative extrema using the First Derivative Test.

(d) Identify the absolute max and min of \( f(x) \) for \( 1 \leq x \leq 3 \).

\[ f'(x) = 3x^2 - 6x = 3x(x - 2) \]

Critical pts \( x = 0, \ x = 2 \)

\( f'(x) < 0 \quad x < 2 \)
\( f'(x) > 0 \quad x > 2 \)

\( f'(0) = -6 \quad f'(2) = 0 \)

Max: \( x = 3 \) \( f(3) = 27 - 27 + 1 = 1 \)

Min: \( x = 2 \) \( f(2) = 8 - 12 + 1 = -3 \)