

Math 329 (Geometry) Exam 1

---

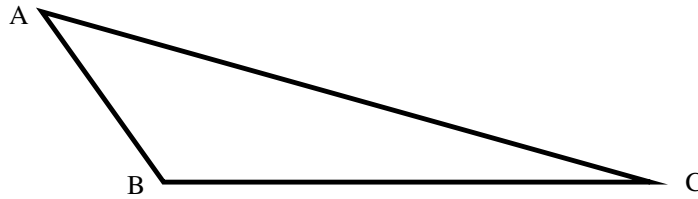
March 5, 2014

Professor Ilya Kofman

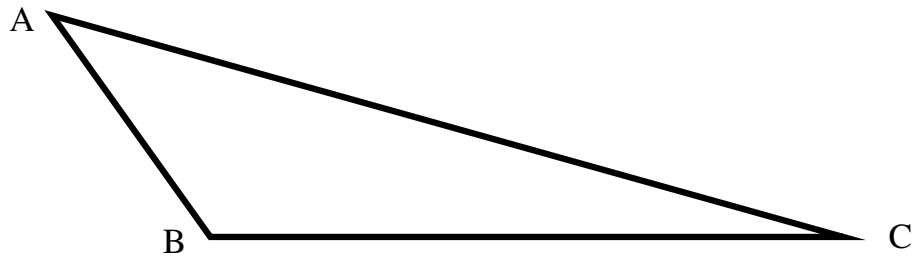
NAME: \_\_\_\_\_

**Problem 1.** Leave all construction marks. Clearly label your steps.

- (a) Construct the circle circumscribed about  $\triangle ABC$ .

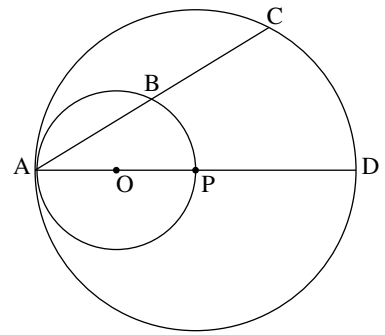


- (b) Construct the circle inscribed in  $\triangle ABC$ .



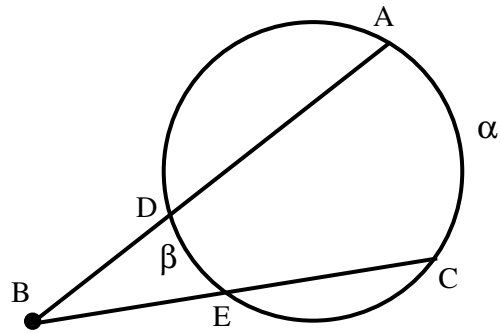
- (c) Given  $AB$  below, construct a regular hexagon with perimeter  $|AB|$ .





**Problem 2.**

In the figure, circles  $O$  and  $P$  are tangent at  $A$ .  
 Show that  $AB = BC$ . (Hint: Draw  $BP$  and  $CD$ .)

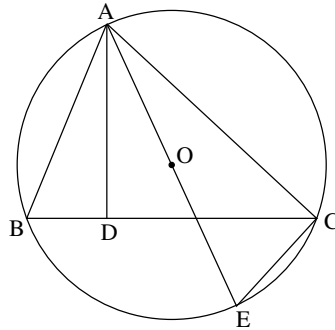


**Problem 3.**

Show that  $m\angle ABC = \frac{1}{2}(\alpha - \beta)$ .  
 Hint: Draw  $AE$ .

**Theorem.** If  $\triangle ABC$  has side lengths  $a$ ,  $b$ ,  $c$ , and  $r$  is the radius of its circumcircle, then  $\text{Area}(\triangle ABC) = \frac{abc}{4r}$ .

**Problem 4.** Complete the proof of this theorem by precisely justifying each step.



Let  $AD$  be an altitude of  $\triangle ABC$ . Let  $O$  be the circumcenter of  $\triangle ABC$ .

(a)  $\angle ABC \cong \angle AEC$

(b)  $\triangle ABD \sim \triangle AEC$

(c)  $AB \cdot AC = AD \cdot AE$

(d)  $\frac{AB \cdot AC \cdot BC}{4r} = \text{Area}(\triangle ABC)$ .

**Problem 5.** On the back, prove the Pythagorean Theorem.