

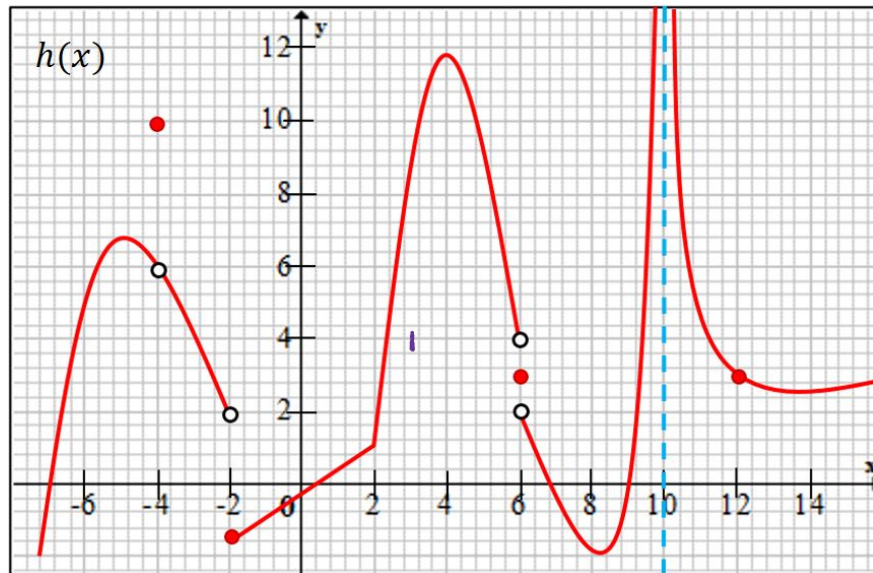
# Business Calculus I (Math 221) Exam 1

September 28, 2016

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Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: \_\_\_\_\_ A



**Problem 1** (20pts). The graph of  $y = f(x)$  is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.

(a)  $\lim_{x \rightarrow -4} f(x) =$

(b)  $\lim_{x \rightarrow 10^-} f(x) =$

(c)  $\lim_{x \rightarrow 2} f(x) =$

(d)  $\lim_{x \rightarrow -2} f(x) =$

(e)  $\lim_{x \rightarrow 6^+} f(x) =$

(f)  $\lim_{x \rightarrow 6^-} f(x) =$

(g) For  $f(x)$  to be continuous at  $x = -4$ , we must set  $f(-4) =$

(h) Estimate the derivative  $f'(0) =$

(i) Estimate the derivative  $f'(4) =$

(j) Estimate the derivative  $f'(12) =$

**Problem 2** (12pts). Evaluate these limits. For an infinite limit, write  $+\infty$  or  $-\infty$ . If a limit does not exist (DNE), you must justify. **Show all work!**

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 11x + 24}{x - 3}$

(b)  $\lim_{x \rightarrow -5} \frac{x^2 + x - 20}{x^2 - 25}$

(c)  $\lim_{x \rightarrow 4^-} \frac{1}{3x - 12}$

(d)  $\lim_{x \rightarrow \infty} \frac{-5x^6 + 2x^2 - 2}{4x^6 + 3x^3 - 2x}$

**Problem 3** (8pts). Recall  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ .

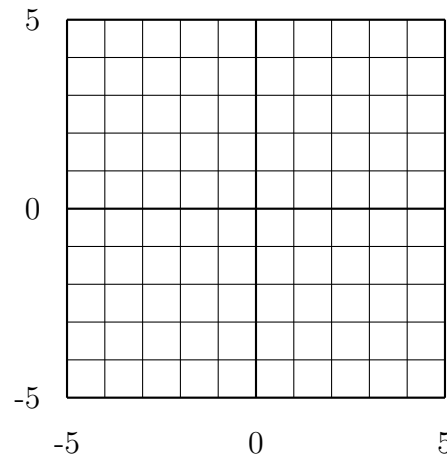
(a) If  $f(x) = 5x^3$ , write the limit for  $f'(2)$ . Do not evaluate this limit.

(b) Show that  $g(x) = |x|$  is not differentiable at 0. Evaluate this limit. **Show all work!**

**Problem 4** (5pts). (a) On the grid below, graph the following piecewise defined function.

$$f(x) = \begin{cases} 3 - 2x & x < 2 \\ x - 5 & x \geq 2 \end{cases}$$

(b) Is the function  $f(x)$  continuous at  $x = 2$ ? (Do not justify.) **YES** **NO**



**Problem 5** (6pts). For what value of  $c$  (if any) is the function  $g(x)$  continuous at  $x = 3$ ? Justify your answer.

$$g(x) = \begin{cases} \frac{8x - 4}{x^2 + 1} & x < 3 \\ c & x = 3 \\ x^2 - 4x + 5 & x > 3 \end{cases}$$

**Problem 6** (24pts). Compute the derivative  $y' = \frac{dy}{dx}$ . Do not simplify. Show all work!

(a)  $y = \frac{x^5}{3} - 4x^{3/4} + 3x + 8 + 15x^{-1/3}$

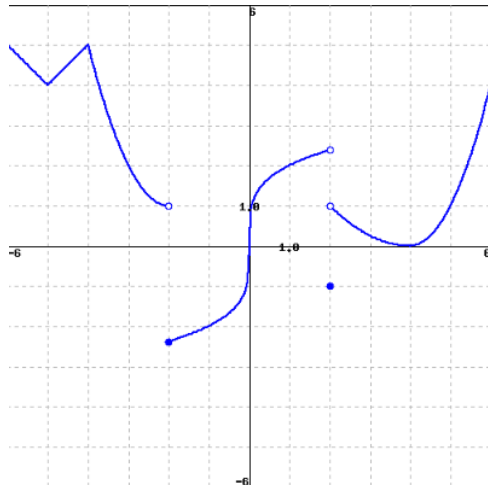
(b)  $y = \frac{7}{\sqrt[3]{x}} - 6\sqrt{x^9} + \frac{12}{x} + \frac{4}{x^7}$

(c)  $y = \sqrt[3]{2x^5 - 3x^2 - 2}$

(d)  $y = \frac{6x^4 + 5x^3}{x^6 - 2}$

(e)  $y = (3x^5 + 4x^4 - 2)(4x^7 - 18)$

(f)  $y = \sqrt{(3x + 2)^4 - 15x}$



**Problem 7** (8pts). The graph of  $y = f(x)$  is shown above for  $-6 < x < 6$ .

(a) For which  $x$  values is  $f(x)$  not continuous?

(b) For which  $x$  values is  $f(x)$  not differentiable?

(c) For which  $x$  values is the derivative  $f'(x) = 0$ ?

**Problem 8** (7pts). Let  $F(x) = 3x^3 - 2x^2 - 10$ . Find the equation of the tangent line to the graph of  $F(x)$  at  $x = 1$ . Leave your answer in the form  $y = mx + b$ .

**Problem 9** (8pts). Let  $g(x) = (2x - 1)^6$ .

(a) Find  $g'(0)$ .

(b) Find  $g''(0)$ .

**Problem 10** (12pts). For  $x$  units sold, the total revenue function is  $R(x) = 30x + 100$ . The total cost function is  $C(x) = 500 + 8x + \frac{1}{8}x^2$ .

(a) Find the profit function  $P(x)$ .

(b) Find the marginal profit when 100 units are sold.

(c) If  $P(100) = 550$ , use your part (b) answer to estimate the total profit if 101 units sold.

(d) Should the company sell the 101st unit? Explain using your answers above.