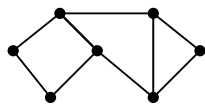# Mathematics for Liberal Arts (Math 102) Exam 1

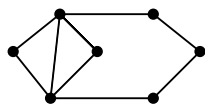Date: February 22, 2007   Professor Ilya Kofman

NAME: _____

Problem 1. *Answer the questions that are below each graph:*

(a) *This graph has an Euler path, which is not a circuit.*   **T   F**

(b) *This graph has an Euler circuit.*   **T   F**

(c) *How many edges will a spanning tree for this graph have?*   _____

(d) *How many edges must be <u>added</u> to best Eulerize this graph?*   _____

(e) *This graph has an Euler path, which is not a circuit.*   **T   F**

(f) *This graph has an Euler circuit.*   **T   F**

(g) *How many edges will a spanning tree for this graph have?*   _____

(h) *How many edges must be <u>added</u> to best Eulerize this graph?*   _____
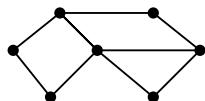
(i) *This graph has an Euler path, which is not a circuit.*   **T   F**

(j) *This graph has an Euler circuit.*   **T   F**

(k) *How many edges will a spanning tree for this graph have?*   _____

(l) *How many edges must be <u>added</u> to best Eulerize this graph?*   _____

Problem 2. *Which <u>one</u> of the following techniques should be applied in each case:*

   I. Apply Kruskal's algorithm to find minimal-cost spanning tree

   II. Find an Euler circuit or best Eulerization

  III. Apply the sorted-edges algorithm to solve TSP

(a) *NYC subway fan wants to quickly*
    visit every station.   **I**   **II**   **III**   **None**

(b) *NYC subway fan wants to quickly*
    see all graffiti on subway tunnel walls.   **I**   **II**   **III**   **None**

(c) *NYC wants to install new expensive fiber*
    optic cable between all subway stations.   **I**   **II**   **III**   **None**

Problem 3. *If a connected graph has 14 vertices of odd valence, at least how many edges must be added to Eulerize the graph?*   _____

Problem 4. *Does the complete graph K_{13} have an Euler circuit? Why or why not?*

Problem 5. *Consider the paths given by the sequences of numbered edges on the graphs as shown. Circle the number of <u>every</u> graph whose path has the property:*

(a) *The path is a circuit.*   **I**   **II**   **III**   **None**

(b) *The path is a spanning tree.*   **I**   **II**   **III**   **None**

(c) *The path covers every edge exactly once.*   **I**   **II**   **III**   **None**

(d) *The path is an Euler circuit.*   **I**   **II**   **III**   **None**

(e) *The path is a Hamiltonian circuit.*   **I**   **II**   **III**   **None**

Problem 6. *For this graph, circle the correct answer below each question.*

(b) *Which routing is produced by the* <u>sorted-edges</u> *algorithm to solve TSP?*

    1) *ABCDA*   2) *ABDCA*   3) *ACBDA*   4) *ACDAB*   5) *ABDAC*

(a) *Which routing is produced by the* <u>nearest-neighbor</u> *algorithm to solve TSP?*

    1) *ABCDA*   2) *ABDCA*   3) *ACBDA*   4) *ACDAB*   5) *ABDAC*

(c) *Which routing is produced by the* <u>brute-force</u> *algorithm to solve TSP?*

    1) *ABCDA*   2) *ABDCA*   3) *ACBDA*   4) *ACDAB*   5) *ABDAC*

(d) *Using Kruskal's algorithm, what is the cost of the spanning tree?*   _____

Problem 7. *Answer the questions for this graph. Show your work for full credit.*

(a) *What is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?*   _____

(b) *What is the cost of the minimal-cost spanning tree using Kruskal's algorithm?*

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