MATH 71300 - FORCING IN SET THEORY

Time and place: Tuesday, 2-4pm in Room 5382
Instructor: Gunter Fuchs
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Content description: The method of forcing was developed by Paul Cohen, who used it to prove the independence of the continuum hypothesis (CH) from the ZFC axioms of set theory, meaning that CH can neither be proved nor refuted based on these axioms. This gave an unexpected answer to Hilbert's first problem on the cardinality of the continuum. Cohen's method is constructive: to show that a statement S is consistent with ZFC, one starts with a model of ZFC and constructs a forcing extension of the original model which satisfies ZFC + S. It was soon developed further to obtain many fascinating results, such as the construction of a model of the ZF axioms (set theory without choice) in which every set of reals is Lebesgue-measurable (starting from a model with an inaccessible cardinal), the independence of Souslin's hypothesis, etc. The use of an inaccessible cardinal here foreshadows many intricate connections between forcing and large cardinals, some of which we will explore, with a focus on iterated forcing and forcing axioms. This course will cover the topics mentioned and is intended to be accessible to students with minimal prior knowledge in logic. Some set-theoretic background will be provided as needed, depending on the audience.

Grading: Homework sets will be distributed regularly, and the grade for the class will be based on the percentage of the cumulative score achieved.

Help: I can reached most easily by email, and I will have some time to discuss questions after class. For students who want to read up on basic logic, I recommend [1]. For background more specifically on set theory, [3] is an almost encyclopedic source. When it comes to the details of forcing, I follow the approach taken in [5] or [6]. A lot of useful information on the interplay between forcing and large cardinals can be found in [4]. The lecture notes for my logic class [2] may also be useful, and they are available on Blackboard.

References

[1] Herbert B. Enderton. *A Mathematical Introduction to Logic*. Academic Press, New York, 1972.
[2] Gunter Fuchs. Set theory and logic. *Notes for a class taught at the CUNY GC in Spring 2022*, 2022.
[3] Thomas Jech. *Set Theory: The Third Millenium Edition, Revised and Expanded*. Springer Monographs in Mathematics. Springer, Berlin, Heidelberg, 2003.
[4] Akihiro Kanamori. *The Higher Infinite*. Springer Monographs in Mathematics. Springer, second edition, 2003.
[5] Kenneth Kunen. *Set Theory. An Introduction To Independence Proofs*. North Holland, 1980.
[6] Kenneth Kunen. *Set Theory*. College Publications, revised edition, 2013.