

Classwork 1

Introduction to Mathematical Proof, MTH 301

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Reading: Its really important in this class that you read the book carefully.
Read Section 1.3.

Problems:

1. (GYHD 3) List explicitly the elements of the truth set for each of these open sentences, using the universes indicated:
 - (a) $0 < x < 10$ (universe = $\mathbf{Z}$).
 - (b) $(t - 2)(t^2 - 9) = 0$ (universe = $\mathbf{R}$).
 - (c) B is a subset of $\{2, 4, 6, 8, 10\}$ with exactly two distinct elements (universe = the set of subsets of $\{2, 4, 6, 8, 10\}$).
2. For each of the following conditional statements, state whether each of the given substitutions represents a counterexample.
 - (a) If n is a prime, then n is odd.
 - i) $n = 2$.
 - ii) $n = 6$.
 - iii) $n = 9$.
 - (b) If x is an even integer, then x is a multiple of 4.
 - i) $x = 5$.
 - ii) $x = 2$.
 - iii) $x = 12$.
 - (c) If $A \subseteq B$, then A and B have an element in common.
 - i) $A = \{1, 2, 3\}$, $B = \{5, 6, 7\}$.
 - ii) $A = \emptyset$, $B = \emptyset$.
 - iii) $A = \{2\}$, $B = \mathbf{Z}$.
 - (d) If $a > b$ and $c > d$, then $ac > bd$.
 - i) $a = 3$, $b = 1$, $c = 8$, $d = 5$.

- ii) $a = 2, b = 4, c = 10, d = -2$.
 - iii) $a = -2, b = -4, c = 5, d = -3$.
- (e) If u is an odd integer, then u is even.
 - i) $u = 0$.
 - ii) $u = 3$.
- 3. For each of the following statements, write an “if $\dots$, then $\dots$ ” statement, introducing appropriate variables, which makes the same assertion (all are intended to be interpreted so that they are true statements):
 - (a) The square of a real number is not negative.
 - (b) Any point that is in the first quadrant has positive coordinates.
 - (c) A proper subset of a set with three elements can’t have more than two distinct elements.
 - (d) All pairs of lines in the plane that aren’t parallel are identical or meet in just one point.
- 4. For each of the following statements, first decide if it is true or false. Then, if it is true, give an informal explanation. If it is false, give a counterexample.
 - (a) Any divisor of an odd integer is odd.
 - (b) Any divisor of an even integer is even.
 - (c) The quotient of two even integers is an even integer.
 - (d) If a, b , and c are real numbers with $a < b$ and $a < c$, then $a < b+c$.
 - (e) If a and b are real numbers with $a > b$, then $a > b - 1$.
 - (f) If a, b , and c are real numbers with $a < bc$, then either $a < b$ or $a < c$.
- 5. Prove the following, using the “Basic Assumptions about Arithmetic” from Appendix B:

If n is a positive integer such that $1/n$ is an integer, then $n = 1$.