Sample Problems for Exam 1
Calculus III, MTH 233, Fall 2020

- Exam 1 will be held in class on Wednesday Oct 7th.
- Review for Exam 1 will be held on Monday Oct 5th.
- The actual Exam will be shorter. We will discuss the format of the Exam during the review.

1. Let \( \vec{a} = \langle 4, -3, -1 \rangle \) and \( \vec{b} = \langle -2, -3, 5 \rangle \). Compute the following:
   (a) \( \vec{a} + \vec{b} \)
   (b) \( 2\vec{a} - 3\vec{b} \)
   (c) \( ||\vec{a} + \vec{b}|| \)
   (d) \( \vec{a} \cdot \vec{b} \)
   (e) \( \vec{a} \times \vec{b} \)
   (f) \( \text{proj}_{\vec{a}} \vec{b} \)

2. (a) Find the area of the triangle with vertices (1, 2, 3), (3, 1, 5) and (4, 5, 6).
   (b) Determine whether the points (1, 1, 2), (0, 1, 0) and (1, 2, 3) are collinear.
   (c) Determine whether the points (0, 2, 1), (0, 1, 0), (1, 1, 0) and (1, 2, 3) are coplanar. (Hint: Use vector triple product.)

3. Find the angle between the planes \( y - z = 5 \) and \( x - z = 7 \).

4. (a) Find the equation of the plane passing through the points (1, 3, 2), (0, 3, 0) and (2, 4, 3).
   (b) Find the equation of a plane passing through point (2, 1, 0) and parallel to the plane \( x - 2y + 5z = 3 \). Does this plane pass through the origin?
   (c) Find the equation of a plane perpendicular to the line \( x = 2 + 3t, y = -t, z = -1 + t \) and passing through point (0, 1, -1).

5. Identify the following surfaces using their traces in planes parallel to the co-ordinate planes.
   (a) \( 9x^2 + 4y^2 = 2z^2 \)
   (b) \( 4x^2 + 4y^2 + z^2 = 16 \)
   (c) \( 4x^2 + 4y^2 - z^2 = 16 \)
   (d) \( 9x^2 - 4y^2 = 72 \)
   (e) \( z = 9x^2 - 4y^2 \)
   (f) \( z = 9x^2 + 4y^2 \)

6. Let \( \vec{r}(t) = \langle \cos 2t, \sin 2t, 3t - 5 \rangle \).
   (a) Compute \( \vec{r}'(t), \vec{r}''(t), \vec{r}'(t) \cdot \vec{r}''(t) \), and \( \int \vec{r}(t)dt \).

7. Let \( \vec{r}(t) = \langle 3 \cos 2t, 3 \sin 2t, 5t \rangle \).
   (a) Find speed at \( t = 2 \).
   (b) Find equation of tangent line at \( t = 2 \).
   (c) Find the arc length of the path for \( 0 \leq t \leq 4 \).
   (d) Find the arc length parametrization for \( \vec{r}(t) \).

8. Compute the following higher partial derivatives.
   (a) Let \( k(x, y, z) = e^{xyz} \). Compute \( k_{xyz}, k_{yxz} \).
   (b) Let \( f(x, y) = \sin(x^2 - y^2) \). Compute \( f_{xx}, f_{xy} \).
9. Compute $\nabla f(1, 2)$ for the following functions:
   (a) $f(x, y) = 4xy^3$  
   (b) $f(x, y) = \ln(x^2 + xy^2)$

10. Find the linearization for the following functions at given points.
    (a) $f(x, y) = x \cos\left(\frac{y}{x}\right)$ at $P = (1, 0)$.
    (b) $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$ at $P = (1, 1)$.

11. Find the equation of the tangent plane to the graph of $f(x, y) = xy^2 - xy + 3x^3y$ at $P(1, 3)$.

12. Suppose the plane $z = 2x - y - 3$ is tangent to the graph of $z = f(x, y)$ at $P(2, 4)$.
    (a) Find $f(2, 4)$, $f_x(2, 4)$, $f_y(2, 4)$.
    (b) Approximate $f(2.2, 3.9)$.

13. Compute the directional derivative at $P$ in the direction $\mathbf{v}$ for the following functions:
    (a) $f(x, y, z) = zx - xy^2$, $P(3, -1)$, $\mathbf{v} = (2, -1, 2)$
    (b) $f(x, y, z) = \sin(xy + z)$, $P(0, 0, 0)$, $\mathbf{v} = \mathbf{j} + \mathbf{k}$

14. Find an equation of the tangent plane at $P(0, 3, -1)$ to the surface
    
    $$ze^x + e^{z+1} = xy + y - 3.$$

15. The temperature at a point in the plane is $T(x, y) = 100 - 3x^2 - 2y^3$. A bug is at the point $(1, -1)$.
    (a) Compute $\nabla T(1, -1)$.
    (b) Find the rate of change of temperature in the direction of $\mathbf{v} = (3, -4)$.
    (c) Find the direction in which the bug should move to increase its temperature the fastest.
    (d) Find a direction in which the bug should move to NOT change its temperature.