

Worksheet on 12.6: Quadric surfaces



Calculus III, MTH 233

Identify the surface by looking at its traces in the co-ordinate planes or planes parallel to them (i.e planes $x = c_1$, $y = c_2$ or $z = c_3$). Write if the traces are ellipses, parabolas or hyperbolas. Refer to the equations of quadric surfaces and conic sections on the other side.

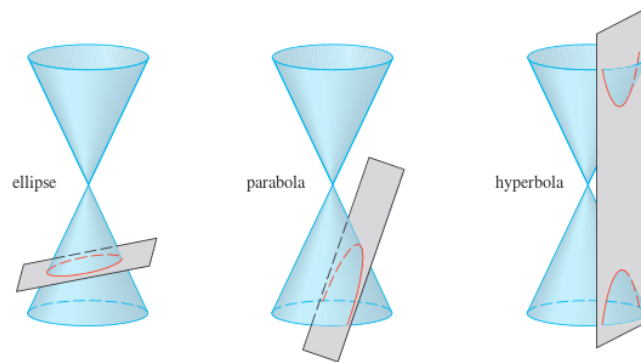
Equation	xy -traces ($z = \text{constant}$)	yz -traces ($x = \text{constant}$)	xz -traces ($y = \text{constant}$)	Surface
$x^2 + y^2 = 2z^2$				
$z = x^2 + 3y^2$				
$x^2 + 4y^2 + 9z^2 = 1$				
$x^2 + 2y^2 = z^2 + 1$				
$x = 2z^2 - y^2$				
$y^2 + 2z^2 = x^2 - 1$				
$y^2 + 4z^2 = 1$				
$x - 3y + 4z = 7$				

Classification of planes and quadric surfaces

Note that the equations are based on z -axis, but they can also be based on x - or y -axis.

Equation	xy -traces ($z = \text{constant}$)	yz -traces ($x = \text{constant}$)	xz -traces ($y = \text{constant}$)	Surface
$ax + by + cz = d$	Lines	Lines	Lines	Plane
$(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$	Ellipses	Ellipses	Ellipses	Ellipsoid
$z = (x/a)^2 + (y/b)^2$	Ellipses	Parabolas	Parabolas	Elliptic Paraboloid
$z = (x/a)^2 - (y/b)^2$	Hyperbolas	Parabolas	Parabolas	Hyperbolic Paraboloid saddle surface
$(x/a)^2 + (y/b)^2 = (z/c)^2 + 1$	Ellipses	Hyperbolas	Hyperbolas	Hyperboloid (one sheet)
$(x/a)^2 + (y/b)^2 = (z/c)^2 - 1$	Ellipses	Hyperbolas	Hyperbolas	Hyperboloid (two sheets)
$(x/a)^2 + (y/b)^2 = (z/c)^2$	Ellipses	Lines	Lines	Cone
$f(x, y) = k$	Level curve of f	Lines	Lines	Cylinder based on level curve

Conic Sections



Ellipse

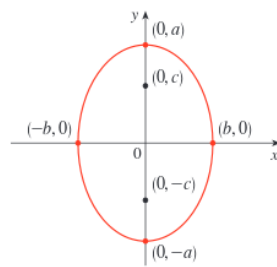


FIGURE 9
 $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a \geq b$

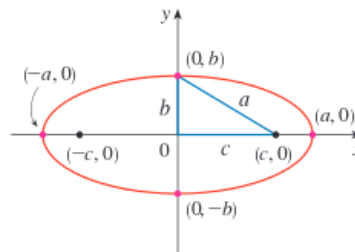
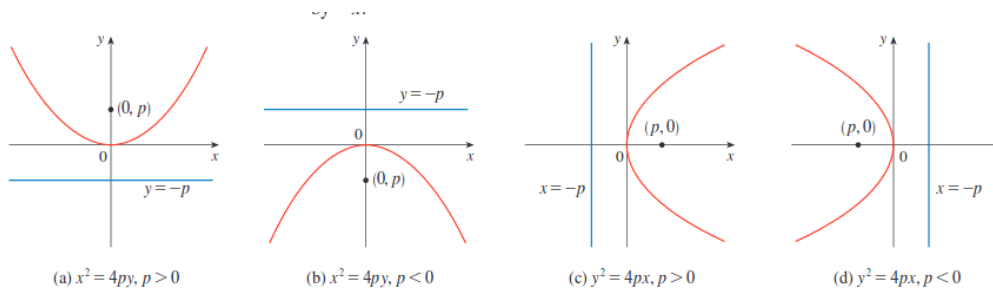


FIGURE 8
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Parabola



Hyperbola

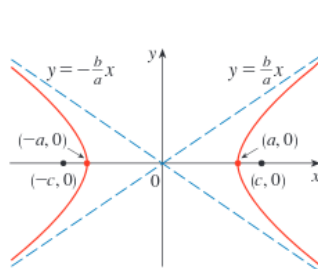


FIGURE 12
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

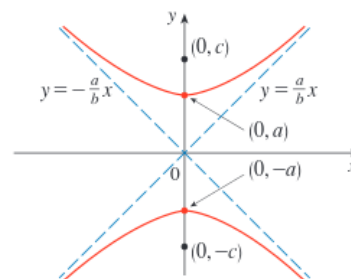


FIGURE 13
 $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$