Solutions to Sample Problems for Exam 3

Calculus I, MTH 231, Spring 2019
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THE CITY UNIVERSITY OF NEW YORK College of Staten Island

- Exam 3 will be held in class on Monday May 13th.

1.

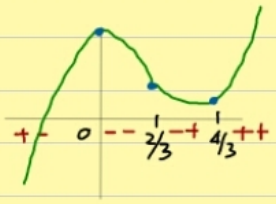
(8) (3)

(a) $f(x)= x^3-2x^2+3$, $f'(x)= 3x^2-4x$, $f''(x)= 6x-4$

critical pts: $3x^2-4x=0$ $x(3x-4)=0 \Rightarrow x=0, x=4/3$ $+\ 0\ -\ 4/3\ +$

infl pts: $6x-4=0 \to x= 2/3$ $-\ 2/3\ +$

Incr: $(-\infty,0), (4/3,\infty)$, Decr: $(0, 4/3)$ CU: $(2/3,\infty)$, CD: $(-\infty,2/3)$



Transition pts: $x= 0, 2/3, 4/3$

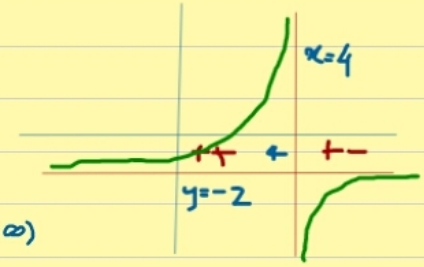
Asymptotes: None

(b) $f'(x)= \frac{6}{(4-x)^2}$, $f''(x)= \frac{12}{(4-x)^3}$

critical pt: $x=4$, f' DNE $+\ 4\ +$

inf pt: none, $x=4$ f'' DNE $+\ 4\ -$

Incr: $(-\infty,4), (4,\infty)$, CU: $(-\infty,4)$ CD: $(4,\infty)$



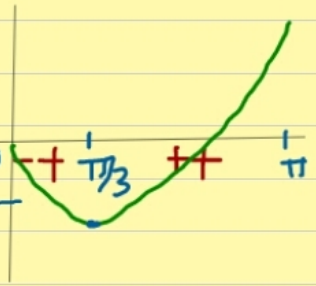
Transition pt: $x=4$

Asymptotes: HA: $y=-2$, VA: $x=4$

(c) $f'(x)= 1-2\cos x$, $f''(x)= 2\sin x$

critical pt: $1-2\cos x=0 \Rightarrow \cos x=1/2$, $x= \pi/3$ (in $[0,\pi]$) $-\ \pi/3\ +$

infl. pts: $2\sin x=0 \Rightarrow x=0,\pi$ (end pts) no infl pts $+\ \pi\ -$



Incr: $(\pi/3,\pi)$, Decr: $(0,\pi/3)$, CU: $(0,\pi)$

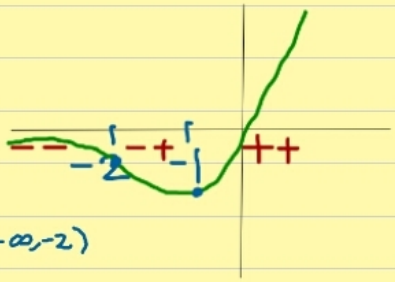
Transition pts: $x= \pi/3$, No asymptotes

(d) $f'(x)= e^x+xe^x= e^x(x+1)$, $f''(x)= e^x+(x+1)e^x= e^x(x+2)$

critical pts: $e^x(x+1)=0 \Rightarrow x=-1$ $-\ -1\ +$

infl. pts: $e^x(x+2)=0 \Rightarrow x=-2$ $-\ -2\ +$

Incr: $(-1,\infty)$, Decr: $(-\infty,-1)$, CU: $(-2,\infty)$, CD: $(-\infty,-2)$



Transition pts: $x=-1,-2$

Asymptotes: VA none, $\lim_{x\to\infty} xe^x = \lim_{x\to\infty} \frac{x}{e^{-x}} = \lim_{x\to-\infty} -\frac{1}{e^{-x}} = 0$, HA: $y=0$

2.

a) $-3/7$ b) 0 c) 0 d) 1

3.

(11) Minimize fence = $x+2y$

Given area = $xy=1000 \Rightarrow y = 1000/x$

$f(x) = x + 2000/x \quad x>0, \quad f'(x) = 1 - \frac{2000}{x^2} = 0 \Rightarrow x^2 = 2000, \quad x = \pm\sqrt{2000}$

As $x>0 \Rightarrow x = \sqrt{2000} = 20\sqrt{5} = 44.72 \quad y = \frac{1000}{\sqrt{2000}} = \sqrt{500} = 22.36$

Dimensions are 44.72×23.66 ft

4.

(12) Let x = base & y = height, Minimize Cost = $\$1(x^2+x^2) + \$2(4xy) = 2x^2 + 8xy$

Given volume = $x^2y = 20\text{ft}^3 \Rightarrow y = 20/x^2$

$f(x) = 2x^2 + 160/x, \quad x>0, \quad f'(x) = 4x - 160/x^2 = 0 \Rightarrow x^3 = 40, \quad x \approx 3.42$

$y = 20/(3.42)^2 = 1.71$, Dimensions are $3.42 \times 3.42 \times 1.71$ ft

5. (a) $R_3 = 65, L_3 = 38$
   (b) $R_6 = 0.7426, L_6 = 0.8259.$

6. $\Delta x = 3/N, a = 0, b = 3.$

$$R_N = \Delta x \sum_{j=1}^{N} f(3j/N) = \frac{3}{N}\sum_{j=1}^{N}(3(\frac{3j}{N})^2 + 2) = \frac{3}{N}\sum_{j=1}^{N}\frac{27j^2}{N^2} + \frac{3}{N}\sum_{j=1}^{N}2) = (\frac{81}{N^3}\sum_{j=1}^{N}j^2) + 6$$

Hence $R_N = \left(\frac{27(N+1)(2N+1)}{2N^2} + 6\right)$

So $\int_0^3 3x^2 - 5x + 2\,dx = \lim_{N\to\infty} R_N = 27 + 6 = 33.$

7. (a) $-\sin(4-7t)/7 + C$ (b) $9t^{13/9} + 21/2t^{2/3} + C$ (c) $2\ln(5u+2)/5 + C$
   (d) $2(4-\cos x)^{3/2}/3 + C$ (e) $14s^{9/2}/9 + 4s^{3/2}/3 + C$ (f) $\sin^4 y/4 + C$
   (g) $e^{x^3}/3 + C$ (h) $(\ln x)^2/2 + C$

8. (a) 2/3 (b) 3/4 (c) $\sqrt{2}-1$ (d) $-49/3$ (e) 1/2
   (f) $\frac{6^{3/2}-3^{3/2}}{3}$ (g) $-1/18$ (h) 0

9. (a) $y' = \sin(x^3)$ (b) $y' = \sin^3 x \cos x$ (c) $y' = -2/x$

10. Done in class during Review.