Solutions to Sample Problems for Exam 2

Calculus I, MTH 231
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1.

1. Use $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

a) $f(x+h) - f(x) = 2(x+h)^2 + 3(x+h) + 1 - (2x^2+3x+1) = 2(x^2+2xh+h^2) + 3(x+h) + 1 - (2x^2+3x+1)$

$= 4xh + 4h^2 + 3h$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (4xh+4h^2+3h)/\cancel{h} = 4x+3$

b) $f(x+h) - f(x) = \frac{2}{x+h+1} - \frac{2}{x+1} = \frac{2(x+1)-2(x+h+1)}{(x+h+1)(x+1)} = -2h/(x+h+1)(x+1)$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} -2\cancel{h}/\cancel{h}(x+h+1)(x+1) = -2/(x+1)^2$

c) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+3}-\sqrt{x+3}}{h}$ rationalize $= \lim_{h\to 0} \frac{(x+h+3)-(x+3)}{(\sqrt{x+h+3}+\sqrt{x+3})h} = 1/2\sqrt{x+3}$

2.

2. a) $3x^2 + 1/2\sqrt{x} + 8/x^5$ b) $(\sin x(e^x+5x^4) - (e^x+x^5)\cos x)/\sin^2 x$

c) $3(x^4-3x^2+5)^2(4x^3-6x)$ d) $(2^x+3)(2x) + (x^2+1)(2^x \ln 2)$

e) $\frac{6x^5}{(1+x^6)^2}$ f) $e^{4x}\sec^2(1+x) + 4e^{4x}\tan(1+x)$ g) $\frac{-2x+\cos x}{x^2+\sin x}$

h) $(1-y^2-2xy)/(2xy+x^2-2)$ i) $7^{1-\cos x} \ln 7 \sin x$ j) $x^{\sqrt{x}}\left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right)$

3.

3. a) $y' = 1 + 1/2\sqrt{x}$, slope $= 3/2$, pt $= (1,2)$, $y = \frac{3x}{2} + \frac{1}{2}$

b) $y' = 2\sin x\cos x + 2(x - \pi/4)$, slope $= 1$, pt $= (\pi/4, 1/2)$, $y = x - \pi/4 + 1/2$

c) $\sin y + x\cos y\, y' - y\sin x + \cos x\, y' = 0$, $y' = \frac{y\sin x - \sin y}{x\cos y + \cos x}$, slope $= 0$, pt $= (\pi/2, 0)$

$y = 0$

4. Done in class. See in book.

5.

6. a) $3 \times 23!$ b) $-27\cos 3x$ c) $f^{(n)}(x) = 5^n e^{5x}$

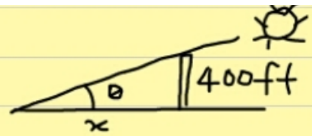
6.

8. $V = x^3$, $A = 6x^2$, $dV/dt = 3x^2\, dx/dt$, $dA/dt = 12x\, dx/dt$

$dV/dt = 75000$ cm^3/min, $dA/dt = 6000$ cm^2/min

7.

9. $d\theta/dt = -0.25$ rad/h, $\tan\theta = 400/x$



$\sec^2\theta \, d\theta/dt = -\frac{400}{x^2}\, dx/dt$, $\frac{dx}{dt} = \frac{x^2\sec^2\theta}{-400}\frac{d\theta}{dt}$

$\frac{dx}{dt} = 400\sqrt{3}$ ft/h

8.

③ a) $v(2) = 48$, $v'(t) = 32 - 8t$, $v'(2) = 32 - 16 = 16$

Linearization $L_a(t) = 16(t-2) + 48 = 16t + 16$

b) $f(1) = e^{-1/2} = 1/\sqrt{e}$, $f'(x) = -x e^{-x^2/2}$, $f'(1) = -e^{-1/2} = -1/\sqrt{e}$

Linearization $L_a(x) = -\frac{1}{\sqrt{e}}(x-1) + \frac{1}{\sqrt{e}} = \frac{-x}{\sqrt{e}} + \frac{2}{\sqrt{e}}$

9.

④ Volume $= V = \frac{4}{3}\pi r^3$, $\Delta V \approx dV = V'(r)dr$, $r=6, dr=0.3$, $V'(r) = 4\pi r^2$

$\Delta V \approx dV = 4\pi 6^2 \times 0.3 = 135.6 \text{ in}^3$

Surface Area $= A = 4\pi r^2$, $\Delta A \approx dA = A'(r)dr$, $r=6, dr=0.3$, $A'(r) = 8\pi r$

$\Delta A \approx dA = 8\pi \times 6 \times 0.3 = 45.2 \text{ in}^2$

10.

⑤ $\sqrt[3]{27.05} - 3 = \sqrt[3]{27.05} - \sqrt[3]{27} = \Delta f$, $f(x) = \sqrt[3]{x}$, $a = 27$, $\Delta x = 0.05$, $f'(x) = 1/3x^{2/3}$

$\Delta f \approx f'(a)\Delta x = \frac{1}{3(27)^{2/3}} \times 0.05 = 0.0055$

11.

⑦ a) $f'(x) = 24x^3 - 24x^5 = 24x^3(1-x^2) = 0 \Rightarrow x = 0, 1, -1$ critical pts

candidates: $-2, 2, 0, 1, -1$ $f(x)$: $-160, -160, 0, 2, 2$

abs max: $f(1) = f(-1) = 2$, abs min: $f(-2) = f(2) = -160$

ⓑ $g'(\theta) = 2\sin\theta\cos\theta + \sin\theta = \sin\theta(2\cos\theta + 1) = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = -1/2$

In $[0, 2\pi]$, $\theta = 0, \pi, 2\pi$, $2\pi/3, 4\pi/3$ critical pts $g(\theta)$: $-1, 1, -1, 5/4, 5/4$

abs max: $g(2\pi/3) = g(4\pi/3) = 5/4$, abs min: $g(0) = g(2\pi) = -1$

5 a) $y' = 3x^2 - 12x = 0$, $x = 0, 4$ b) $y' = \cos x + \sin x = 0$, $\tan x = -1$, $x = -\pi/4 + n\pi$

c) $y' = x + 2x\ln x = 0$, $x = e^{-1/2}, 0$

12.

Discussed in clas after Exam 1.

13.

10.

14.

(c) Eqn of tangent line: Slope $= \frac{4-2}{10-4} = \frac{1}{3}$, point $= (4,2)$

Eqn $y-2 = \frac{1}{3}(x-4)$, $y = \frac{x}{3} + \frac{2}{3}$, so linearization $L(x) = \frac{x}{3} + \frac{2}{3}$

$f(4.55) \approx L(4.55) = \frac{4.55}{3} + \frac{2}{3} = 2.1833$

15. Using Implicit differentiation $y' = -4x/y$. Since tangent is parallel to $y = 2x + 10$, $y' = -4x/y = 2 \implies y = -2x$. In order to get the points, we need to substitute this back in the equation i.e.

$$4x^2 + (-2x)^2 = 8 \implies 8x^2 = 8 \implies x^2 = 1 \implies x = \pm 1.$$

Since $y = -2x$, this gives$x = 1, y = -2$ and $x = -1, y = 2$. The points are $(1, -2)$ and $(-1, 2)$.