Sample Problems for Exam 2

Calculus I, MTH 231, Spring 2019
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- Exam 2 will be held in class on Wednesday April 3rd. Review will be held on Monday April 1st.
- Syllabus for Exam 2: Chapters 3, 4.1, 4.2.
- Solutions to Webwork problems on Chapter 3 will be posted on March 30th.
- Best way to prepare for the midterm is to read the book, solve the sample problems and webwork problems.

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1. Write the definition of derivate $f'(x)$ of $f(x)$. Compute the derivatives of the following functions **using the definition of the derivative**.

   (a) $f(x) = 2x^2 + 3x + 1$ (b) $f(x) = \dfrac{2}{x+1}$ (c) $f(x) = \sqrt{x+3}$

2. Calculate y'.

   (a) $y = x^3 + \sqrt{x} - \dfrac{2}{x^4}$

   (b) $y = \dfrac{e^x + x^5}{\sin x}$

   (c) $y = (x^4 - 3x^2 + 5)^3$

   (d) $y = (2^x + 3)(x^2 + 1)$

   (e) $y = \tan^{-1}(x^6)$

   (f) $y = e^{4x}\tan(1+x)$

   (g) $y = \ln(x^2 + \sin x)$

   (h) $xy^2 + x^2y = x + 2y$

   (i) $y = 7^{1-\cos x}$

   (j) $y = x^{\sqrt{x}}$

3. Find the equation of tangent to the given curve at the given point.

   (a) $y = x + \sqrt{x}$ at $x = 1$ (b) $y = \sin^2 x + (x - \pi/4)^2$ at $x = \pi/4$
   (c) $x\sin y + y\cos x = 0$ at $(\pi/2, 0)$

4. Use implicit differentiation to compute the derivative of the following inverse functions.

   (a) $y = \sin^{-1} x$ (b) $y = \tan^{-1} x$ (c) $y = \ln x$

5. (a) $f(x) = 3x^{237} + 5x^{123} - 7$. Find $f^{(237)}(x)$.

   (b) $f(x) = \sin(3x)$. Find $f'''(x)$.

   (c) $f(x) = e^{5x}$. Guess $f^{(n)}(x)$ by computing first few derivatives.

6. The side of a cube is increasing at the rate of 10 cm^3/min. Find the rate at which the volume and surface area of the cube is increasing when the radius is 50 cm.

7. The angle of elevation of the Sun is decreasing at the rate of 0.25 rad/h. How fast is the shadow cast by a 400 ft tall building increasing when the angle of elevation of the Sun is $\pi/6$?

8. Find the linearization of the function at the given point.
   (a) $v(t) = 32t - 4t^2$, $a = 2$ (b) $f(x) = e^{-x^2/2}$, $a = 1$

9. A spherical balloon has a radius of 6 inches. Use differentials to estimate the change in volume and surface area if the radius increases by 0.3 in.

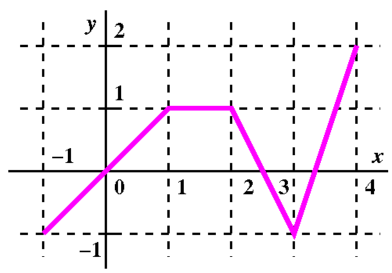
10. Use linear approximation to approximate $\sqrt[3]{27.05} - 3$.

11. Find critical points and extreme values of the following functions on the given intervals.

    (a) $f(x) = 6x^4 - 4x^6$ on $[-2, 2]$
    (b) $g(\theta) = \sin^2\theta - \cos\theta$ on $[0, 2\pi]$
    (c) $y = \sin x - \cos x$ on $[0, 2\pi]$.
    (d) $y = x^2 \ln x$
    (e) $y = x^3 - 6x^2$

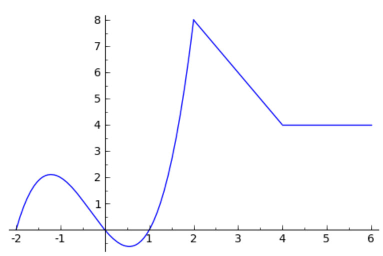
12. The graph of $y = f(x)$ is given below.

    Find $f'(0), f'(1), f'(1.5)$, $f'(2.5), f'(3), f'(3.5)$



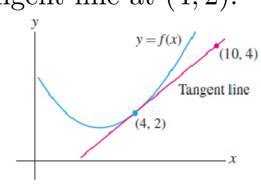
13. The graph of $y = f(x)$ is given below.

    Indicate the points which have horizontal tangents, the points where $f(x)$ is not differentiable and sketch the graph of $f'(x)$ (on the same graph).



14. * The graph of $y = f(x)$ is given along with its tangent line at $(4, 2)$.

    Use the graph to approximate $f(4.55)$.



15. * Find the points on the curve $4x^2 + y^2 = 8$ where the tangent line is parallel to the line $y = 2x + 10$.