Sample Problems for Exam 1

Calculus I, MTH 231, Spring 2019
Instructor: Abhijit Champanerkar

THE CITY UNIVERSITY OF NEW YORK
College of Staten Island

- Exam 1 will be held in class on Wednesday Feb 27th.
- Review for Exam 1 will be held on Monday Feb 25th.
- Syllabus for Exam 1: Sections covered from Chapters 1, 2, 3.1, 3.2.
- Best way to prepare for the midterm is to solve the Classworks, Sample problems and Webwork Problems.

---

1. (a) Find the equation of the line passing through points $(3, -4)$ and $(5, 1)$. Is the point $(2, -1)$ on this line ?

   (b) Find the equation of the line passing through point $(1, 2)$ and parallel to the line $4x - 2y = 3$.

2. Find the functions $f \circ g,\ g \circ f,\ f \circ f, g \circ g$ where $f(x) = \cos x$ and $g(x) = x^2 - 9$.

3. Compute the following limits if they exist.

   (a) $\displaystyle\lim_{x\to -1} \frac{x^2 - x - 2}{x^2 + 3x + 2}$

   (b) $\displaystyle\lim_{t\to 9} \frac{\sqrt{t} - 3}{t - 9}$

   (c) $\displaystyle\lim_{h\to 0} \frac{(4+h)^2 - 16}{h}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\sin 5x \sin 3x}{2x^2}$

   (g) $\displaystyle\lim_{t\to 0} \frac{(t+3)\sin 3t}{5t}$

   (h) $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h}$

   (i) $\displaystyle\lim_{x\to 0} \frac{\tan 7x}{2x}$

   (j) $\displaystyle\lim_{x\to 0} \frac{(x^2-2)(1-\cos 2x)}{4x}$

4. Compute the following limits at infinity.

   (a) $\displaystyle\lim_{x\to \infty} \frac{3x^2 - x - 2}{5x^2 + 3x + 2}$

   (b) $\displaystyle\lim_{t\to -\infty} \frac{3t^3 - 7t + 5}{4t^5 - 13}$

   (c) $\displaystyle\lim_{s\to -\infty} \frac{s^5 + 3s^3 + s}{1 - 5s^2}$

   (d) $\displaystyle\lim_{h\to -\infty} \frac{\sqrt{2h^2+1}}{3h - 1}$

5. Show that $\displaystyle\lim_{x\to -1} \frac{|x+1|}{x+1}$ does not exist.

6. Let

$$f(x) = \begin{cases} ax^2 + 1 & \text{if } x \le 2 \\ x - 4 & \text{if } x > 2 \end{cases}$$

   Find the value of a if $f(x)$ is continuous for all real numbers.

7. Let

$$f(x) = \begin{cases} \sqrt{|x|} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

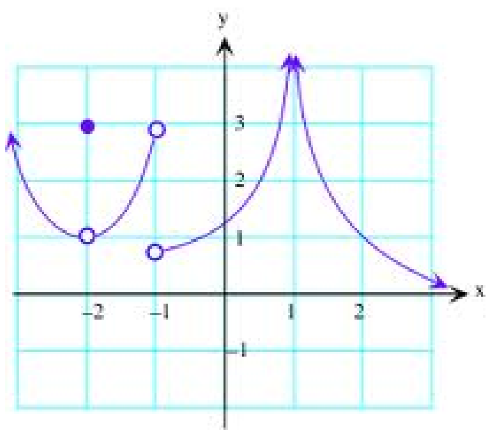
Evaluate each limit if it exists.

(i) $\lim\limits_{x\to 0^+} f(x)$ (ii) $\lim\limits_{x\to 0^-} f(x)$ (iii) $\lim\limits_{x\to 0} f(x)$

(iv) $\lim\limits_{x\to 3^-} f(x)$ (v) $\lim\limits_{x\to 3^+} f(x)$ (vi) $\lim\limits_{x\to 3} f(x)$

Where is f discontinuous ?

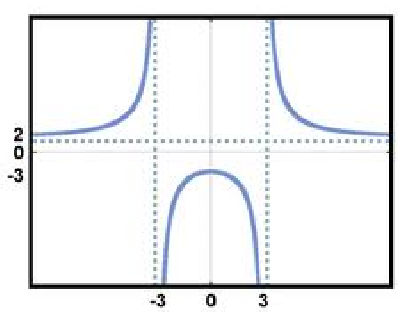
8. The graph of $y = f(x)$ is given below.



(a) Find $\lim\limits_{x\to -2^+} f(x)$, $\lim\limits_{x\to -2^-} f(x)$, $\lim\limits_{x\to 1^+} f(x)$, $\lim\limits_{x\to 1^-} f(x)$, $\lim\limits_{x\to -1^-} f(x)$, $\lim\limits_{x\to -1^+} f(x)$

(b) Where is f discontinuous and why ?

9. The graph of $y = g(x)$ is given below.



(a) Find $\lim\limits_{x\to -3^+} g(x)$, $\lim\limits_{x\to -3^-} g(x)$, $\lim\limits_{x\to 3^+} g(x)$, $\lim\limits_{x\to 3^-} g(x)$, $\lim\limits_{x\to -\infty} g(x)$, $\lim\limits_{x\to \infty} g(x)$

(b) Find the vertical and horizontal asymptotes of g.

10. Compute the derivative using the definition of the derivative.

(a) $f(x) = 2x^2 + 3x + 1$

(c) $f(x) = \sqrt{x+3}$

(b) $f(x) = \dfrac{2}{x+1}$

(d) $f(x) = 3x - 5$

11. Calculate y'.

(a) $y = x^3 + 3x + \sqrt[3]{x}$

(c) $y = \frac{x^4 - 3x^2 + 5}{x^2}$

(b) $y = \dfrac{x^5 + 4}{\sqrt{x}}$

(b) $y = e^x + 1 + x^2$

12. Find the equation of tangent to the given curve at the given point.
(a) $y = x + \sqrt{x}$; $(1, 2)$ (b) $y = (x-3)(x+5)$; $(2, -7)$

13. Find the x-coordinate of the points where the tangent to the given curve is horizontal.

(a) $y = x^3 - 6x^2$

(b) $y = x(3x^2 + 12x - 20)$

14. Find the points on the curve $y = e^x$ where the tangent is parallel to the line $x - 4y = 1$.

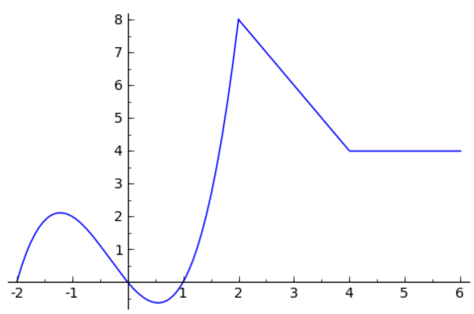
15. Find f and a so that the given limits are $f'(a)$

(a) $\displaystyle\lim_{x \to 0} \frac{\sin(5x)}{2x}$ (c) $\displaystyle\lim_{t \to 1} \frac{t^{500} - 1}{t - 1}$

(b) $\displaystyle\lim_{x \to 0} \frac{\tan x}{3x}$ (d) $\displaystyle\lim_{x \to 2} \frac{3^x - 9}{x - 2}$

16. The graph of $y = f(x)$ is given below.

Indicate the points which have horizontal tangents, the points where $f(x)$ is not differentiable and sketch the graph of $f'(x)$ (on the same graph).



17. Write down precise definitions of the following:

(a) Function

(b) Continuity of $f(x)$ at $x = a$.

(c) Derivative of $f(x)$ at $x = a$.

(d) Vertical asymptote to $y = f(x)$.

(e) Horizontal asymptote to $y = f(x)$.