Recent developments in minimal surfaces

Titles and Abstracts

The Graduate Center, CUNY
365 Fifth Avenue
New York, NY 10016
Science Center, Room 4102

Thursday, February 23rd, 2012
9:00am till 4:00pm

9:30–10:00:	*Coffee*
10:00–11:00:	William P. Minicozzi
11:15–12:15:	Mike Wolf
12:15–1:30:	*Lunch*
1:30–2:30:	Jean-Marc Schlenker
2:45–3:45:	William Meeks
3:45–4:15:	*Discussion*

William Meeks III, University of Massachusetts
Constant mean curvature spheres in homogeneous three dimensional manifolds

Using the classical holomorphic quadratic Hopf differential, Hopf proved that a constant mean curvature $H > 0$ sphere in three-dimensional Euclidean space is a round sphere of radius $1/H$. In particular, the moduli space of such spheres up to congruence is parametrized by the mean curvatures that lie in the interval $(0, \infty)$ and every such sphere has index one for the stability operator. In recent years, this result of Hopf has been generalized to some other simply connected homogeneous 3-manifolds X with 4 dimensional isometry group such as the Riemannian product of a round sphere with the real number line $\mathbb{R}$. I will discuss the final version of this classification result due to Meeks, Mira, Perez and Ros. In my talk I will focus on the case where X is a metric Lie group (a Lie group with a left invariant metric). When X is diffeomorphic to R^3, these spheres of constant mean curvature H are parametrized by their values of H in the open interval $(2Ch(X), \infty)$, where $Ch(X)$ is the Cheeger constant of X, and we show how to calculate $Ch(X)$ in terms of the metric Lie algebra of X. When X is diffeomorphic to the 3-sphere, then we prove that it admits for every H greater than or equal to 0, a unique sphere $S(H)$ of constant mean curvature H, and this sphere has index one and is Alexandrov embedded (in general when H is not equal to 0, $S(H)$ may possibly not be embedded for certain homogeneous metrics). Also when X is diffeomorphic to the 3-sphere, the minimal sphere $S(0)$ in X is embedded and contains 3 geodesics of rotational symmetry that meet orthogonally on $S(0)$, and so $S(0)$ always separates X into isometric regions.

William Minicozzi II, Johns Hopkins University
Dynamics and singularities of mean curvature flow

Mean curvature flow (MCF) is a nonlinear heat equation where the hypersurface evolves to minimize its surface area. Minimal surfaces are static solutions of MCF. If we start the flow at any closed hypersurface in R^n, then singularities must develop. The major problem is to understand the possible singularities and the behavior of the flow near a singularity. I will survey work on this with Toby Colding.

Jean-Marc Schlenker, Université Toulouse III
Maximal surfaces in the anti-de Sitter space and the universal Teichmüller space

A homeomorphims of the circle is quasisymmetric if it extends to a quasiconformal diffeomorphism of the disk. We prove that it then has a unique quasiconformal extension to the hyperbolic disk which is minimal Lagrangian (it is area-preserving and its graph is minimal). The proof is based on an existence and uniqueness result for maximal surfaces in the anti-de Sitter space spanning a given space-like curve at infinity. Joint work with Francesco Bonsante (Pavia).

Michael Wolf, Rice University
Polynomial Pick forms for affine spheres over the complex plane

Convex real projective structures on surfaces, corresponding to discrete surface group representations into $SL(3, R)$, have associated to them affine spheres which project to the convex hull of their universal covers. Such an affine sphere is determined by its Pick (cubic) differential and an associated Blaschke metric; closely related is a minimal surface in the symmetric space $SL(3, R)/SO(3, R)$. As a sequence of convex projective structures leaves all compacta in its deformation space, a subclass of the limits is described by polynomial cubic differentials on affine spheres which are conformally the complex plane and some special minimal surfaces. We develop a theory of these affine spheres which are conformally the plane. Joint work with David Dumas.

Organizers:

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