

# Hyperbolic conservation laws and applications

## Schedule and Abstracts

The Graduate Center, CUNY  
365 Fifth Avenue  
New York, NY 10016  
Science Center, Room 4102

Thursday, April 26th, 2012  
9:30am till 5:30pm

9:30–10:00: *Coffee*

10:00–11:00: Philippe G. LeFloch

11:15–12:15: Yuxi Zheng

12:15–1:30: *Lunch*

1:30–2:30: Barbara Lee Keyfitz

2:45–3:45: Kris Jenssen

3:45–4:15: *Coffee break*

4:15–5:15: Alberto Bressan

## **Alberto Bressan**

Department of Mathematics, Penn State University

*Nash equilibria for traffic flow on networks*

Adopting the classic Lighthill–Whitham model, traffic flow on a network of roads can be described by a family of scalar conservation laws, one for each arc of the network.

It is assumed that individual drivers choose their departure time in order to minimize a cost functional, depending both on their departure and their arrival times. By a “Nash equilibrium” we mean a solution where no driver can lower his individual cost, by changing his own departure time or by selecting a different route to reach the desired destination.

Some existence and uniqueness results for Nash equilibria will be given. The problem of stability of such equilibria will also be discussed.

## **Kris Jenssen**

Department of Mathematics, Penn State University

*TVD fields for pairs of conservation laws and the  $p$ -system*

We consider the issue of large variation solutions to compressible, isentropic flow in one space dimension. Our analysis helps explain why this remains an open problem, almost 50 years after Glimm’s theorem on global existence of small variation solutions. Specifically, we

show that there is no scalar field whose variation along all entropic solutions decay in time. One part of the work provides the general form of such “TVD-fields” for general  $2 \times 2$ -systems. This is joint work with Geng Chen (Postdoc, Penn State).

### **Barbara Lee Keyfitz**

Department of Mathematics, The Ohio State University  
*Using geometric singular perturbation theory to understand singular shocks*

There are classes of conservation laws which do not possess Riemann solutions of the standard type (composed of shocks, rarefactions and linear waves), even in regions where the equations are strictly hyperbolic and genuinely nonlinear. This is a “large data” phenomenon. For some systems, candidates for solutions of lower regularity, now called singular shocks, have been postulated. By means of singular shocks, Riemann problems can be resolved. However, it is unclear in what sense singular shocks satisfy the conservation law. In this talk, I will describe work by Stephen Schecter which uses Geometric Singular Perturbation Theory (GSPT) to prove that there are approximations to singular shocks that satisfy the self-similar Dafermos-DiPerna regularization, at least for a model system. In addition to demonstrating a mechanism for the approximation, GSPT also demonstrates the detailed structure of singular shock profiles. The classic

model system, which gave rise to the discovery of singular shocks, consists of the gas dynamics equations with the wrong variables conserved. In ongoing work with Charis Tsikkou and Ting-Hao Hsu, we are applying this theory to a recent model in chromatography, developed by Marco Mazzotti.

## **Philippe G. LeFloch**

Laboratoire Jacques-Louis Lions

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<http://philippefloch.org>

*Existence results for the Euler equations of compressible fluids in one space dimension*

I will talk about a joint work with P. Germain (NYU) on the Euler equations of compressible fluid flows in one space dimension when the (real) fluid is governed by a general equation of state. I will present existence and compactness results for weak solutions satisfying entropy inequalities, and will discuss finite energy solutions with vacuum states satisfying higher-integrability properties. The mathematical entropy–entropy flux pairs of the Euler equations are described by a fundamental kernel with limited regularity which we determine by a nonlinear superposition formula. We assume solely a very weak regularity assumption on the singularity of the pressure function near the vacuum. Singular products are dealt with in

the analysis of Tartar's commutation relation for Young measures with unbounded support.

**Yuxi Zheng**

Department of Mathematical Sciences, Yeshiva University

*A partial hodograph transform at a sonic curve for the Euler system*

We deal with a patch of solutions to the compressible 2-dimensional Euler system near a sonic curve. A transformation is introduced to split certain singularity from regularity so as to allow us to build a patch of solution that contains singularity.

**Organizers:**

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