Reading

1. Splitting Lemma, page 147.
2. Homology of groups and Proposition 2.45, pages 148 - 149.
3. Discussion and example about last part of the proof of Theorem 2A.1 \((H_1(X) \text{ and } \pi_1(X))\), pages 167-168.
5. (Homological Algebra) Lemmas 3.1 (page 194), 3A.1 (\(\_ \otimes G\) is right exact), 3A.2 and 3A.5 (Properties of Tor), pages 262 - 265.
6. Cohomology of Spaces - cohomological analogs of theorems we saw for homology, pages 197 - 204.

Problems

1. (Left exactness of Hom) For an abelian group \(A\), denote by \(A^* = \text{Hom}(A, G)\). If \(A \rightarrow B \rightarrow C \rightarrow 0\), then show that \(A^* \leftarrow B^* \leftarrow C^* \leftarrow 0\) is exact.
2. Show that \(\text{Tor}(A, B)\) is always a torsion group.
3. Use the Universal Coefficient Theorem to show that if \(H_\ast (X; \mathbb{Z})\) is finitely generated, then for any coefficient field \(F\), we have \(\chi(X) = \sum_n (-1)^n \dim H_n(X; F)\).
4. Using the Universal Coefficient Theorems (or using the definition), compute the homology and cohomology groups with the given coefficient groups (assume homology with \(\mathbb{Z}\) coefficients):
   (a) \(M_g\), the closed orientable surface of genus \(g\) with coefficients \(\mathbb{Q}, \mathbb{Z}, \mathbb{Z}_2, \mathbb{Z}_4\).

\(^1\)All section, chapter, page and example numbers refer to the book “Algebraic Topology” by Allen Hatcher freely available at http://www.math.cornell.edu/~hatcher/AT/ATpage.html
(b) \(N_g\), the closed non-orientable surface of genus \(g\) with coefficients \(\mathbb{Q}, \mathbb{Z}, \mathbb{Z}_2, \mathbb{Z}_4\).

(c) \(\mathbb{R}P^n\) with coefficients \(\mathbb{Q}, \mathbb{Z}, \mathbb{Z}_3\) and \(\mathbb{Z}_4\).

5. Compute the cohomology groups of \(X = \{p\}\) using the definition.

6. Show that a degree \(d\) map of \(S^n\) induces an automorphism of \(H^n(S^n; G)\) given by multiplication by \(d\).

7. Show that for any map \(f : T \to K\), the induced map \(f_* : H_2(T; \mathbb{Z}_2) \to H_2(K; \mathbb{Z}_2)\) is trivial.

8. Let \(C\) be a free chain complex. Show that if \(H_i(C)\) is finitely generated for all \(i\), then \(H_p(C; \mathbb{Z}_n) \cong H^p(C; \mathbb{Z}_n)\).

9. Page 204 - 206: Problems 4, 5, 6, 8, 10

**Hand-in:** 3, 5, 6, 7, 8