Homework 1

Topology I, Math 70800, Spring 2021
Instructor: Abhijit Champanerkar
Due: Monday Feb 15th
(upload pdf scan on CUNY BlackBoard in Homework Section)
Topic: Cell complexes and surfaces

Reading [1]

1. Reading from Chapter 0:
   (a) Basic definitions.
   (b) Page 4, House with 2 rooms.
   (c) Pages 5-8, Cell complexes and examples of the real and complex projective space $\mathbb{R}\mathrm{P}^n$ and $\mathbb{C}\mathrm{P}^n$.
   (d) Pages 11-12, Collapsing subspaces, Examples 0.7, 0.8 and 0.9.
   (e) Pages 14-16, Homotopoy extension property and applications to cell complexes.
2. Read at least one proof of classification of surfaces, available from:
   http://www.math.csi.cuny.edu/abhijit/70800/surfaces/
3. If required please read the examples on Quotient Spaces which are posted on the class homepage.

Problems

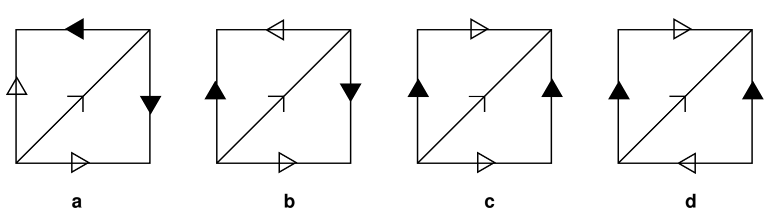
1. Let X be the space obtained from S^2 by attaching n 2-cells along any collection of n disjoint circles in S^2. Show that $X \simeq \bigvee_{n+1} S^2$.
2. Let G be a graph with v vertices and e edges. (a) Show that $G \simeq \bigvee_n S^1$ and find n in terms of v and e. (b) Show that if $G \simeq \bigvee_m S^1$, then $m = n$.

A compact surface S with boundary is a 2-manifold with boundary. Let $\overline{S}$ be the closed surface obtained by gluing disks to the boundary of S. Then the genus of S is the genus

[1] All section, chapter, page and example numbers refer to the book "Algebraic Topology" by Allen Hatcher.

of the closed surface $\overline{S}$. You can model a surface with boundary as the standard surface model with the interiors of disjoint discs deleted.

3. (a) Find the genus of the cylinder and the Mobius strip.
   (b) Show that the torus with a disc removed (punctured torus) is homotopic to the figure eight.
   (c) Let S be a compact surface with boundary with genus g and b boundary components. Show that $S \simeq \bigvee_n S^1$ and find n in terms of g and b.
4. Identify the following surfaces from the given surface symbols.
   (a) $abca^{-1}b^{-1}c^{-1}$ (b) $abca^{-1}db^{-1}c^{-1}d^{-1}$ (c) $ae^{-1}a^{-1}bdb^{-1}cc$
5. Let X be a CW complex.
   (a) Show that attaching a 2-cell to X adds a relation to $\pi_1(X)$.
   (b) Show that attaching an n-cell to X for $n \geq 3$ does not change the fundamental group.
6. (a) Show that the space obtained by identifying the edges of a polygon in pairs is a closed surface.
   (b) Show that the space obtained by identifying pairs of edges of a finite collection of disjoing 2-simplicies is a closed surface.
   (c) Show that the edges in (b) can always be oriented so as to define a Δ-complex structure on the resulting surface. (harder!)
7. Find a Δ-complex structure and use it to compute the simplicial homology for the following spaces:
   (a) $X = \vee_n S^1$ (b) $X = \vee_n S^2$
   (c) X obtained by identifying the verticies of Δ^2 to a point.
8. (a) Identify the surfaces given below, and (b) Compute their simplicial homology.



Hand-in: Problems 3c, 5, 7b, 8(a).