Homework 4

Complex Analysis, MTH 431, Spring 2014

1. Page 89: 5.3, 5.4
2. Page 98: 5.7
3. Page 102: 5.10, 5.11
4. $\int_\gamma \log z \, dz$ where γ is the semi-circle joining $-i$ to i lying in right half-plane Re(z) ≥ 0.
5. Let $\gamma_1, \gamma_2, \gamma_3$ be the following three paths from 0 to $1+i$:
   $\gamma_1(t) = t + it, 0 \leq t, \leq 1,$
   $\gamma_2(t) = t + it^2, 0 \leq t, \leq 1,$
   $\gamma_3(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 + i(t-1) & 1 \leq t \leq 2 \end{cases}$
   Evaluate $\int_{\gamma_i} f(z) \, dz$, $i = 1, 2, 3$ for the following functions. Use any theorems we have learned to reduce computations.
   (a) $f(z) = 2\overline{z} + 1$
   (b) $f(z) = 2z + 1$
   (c) $f(z) = e^z$
6. Let $g, h : [a, b] \to \mathbb{C}$ be continuous functions and $c = \alpha + i\beta$. Prove the following statements.
   (a) $\int_a^b (g(t) + h(t)) \, dt = \int_a^b g(t) \, dt + \int_a^b h(t) \, dt$
   (b) $c \int_a^b g(t) \, dt = \int_a^b cg(t) \, dt$
7. Let $\gamma : [a, b] \to \mathbb{C}$ be a path and f_1 and f_2 be complex functions whose domain contains $[\gamma]$. Use the above exercise to prove the following statements:
   (a) $\int_\gamma (f_1(z) + f_2(z)) \, dz = \int_\gamma f_1(z) \, dt + \int_\gamma f_2(z) \, dz$
   (b) $c \int_\gamma f_1(z) \, dz = \int_\gamma cf_1(z) \, dz$
8. Show that $|\displaystyle\int_\gamma \frac{e^z}{z} \, dz| \leq 2\pi e$ where γ is the unit circle with standard parametrization.

Hand-in Problems Due: Monday March 24thth 2014

1. Page 98: 5.7 a, d
2. Page 102: 5.11
3. Problem numbers 4, 5a, 7b