Homework 3

Complex Analysis, MTH 431, Spring 2014

1. Page 69-70: 4.7, 4.8, 4.9, 4.10, 4.12

2. Find radius of convergence of the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} 2nz^{2n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{z^{2n+1}}{n!}$

   (c) $\displaystyle\sum_{n=0}^{\infty} 5^n z^{3n}$

3. Let P be a non-zero polynomial. Show that the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} P(n)z^n$ is 1.

4. Find the poles of the following functions, and determine their orders.

   (a) $f(z) = \dfrac{1}{(z^2+1)^3(z-1)^4}$

   (b) $f(z) = \dfrac{\sin z}{z^5}$

   (c) $f(z) = \dfrac{1}{1-e^z}$

5. Show that if $f(z)$ has an essential singularity at $z = z_0$ then $1/f(z)$ also has an essential singularity.

6. Show that if $f(z)$ has a zero of multiplicity m at a then $1/f(z)$ has a pole of order m at $z = a$.

7. Find principal values of the following.

   (a) $\log i$

   (b) $\log(1+i)$

Hand-in Problems **Due:** Monday March 3rd 2014

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