Key

Exponential Growth and Decay Word Problems

Write an equation for each situation and answer the question.

(1) Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

$$A = 1 \cdot 2^{t}$$

$$A = 1 \cdot 2^{24}$$

$$\boxed{A = 16{,}777{,}216 \text{ bacteria}}$$

(2) Find a bank account balance if the account starts with \$100, has an annual rate of 4%, and the money left in the account for 12 years.

$$P(t) = 100 \cdot (1 + .04)^{12}$$

$$\boxed{P(t) = \$160.10}$$

(3) In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$$P(t) = 285 \cdot (1.75)^{1994-1985}$$

$$= 285 \cdot (1.75)^{9}$$

$$\approx 43871.99 \qquad \boxed{43{,}872 \text{ subscribers}}$$

(4) The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

a.) 6191

b.) 4%

(5) You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

$$P(t) = 30{,}000\,(1.05)^{2011-1960}$$

$$= 30{,}000\,(1.05)^{51}$$

$$= \boxed{\$361{,}223.09}$$

(6) During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

$y = t$

$$P(t) = 500 \cdot (1 - .12)^t$$
$$= 500 \cdot (.88)^{240}$$
$$= \boxed{2.37 \times 10^{-11}\ mL}$$

(7) An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

$$P(t) = 400 \cdot (1 - .29)^6$$
$$= 400(.71)^6$$
$$= \boxed{51.24\ mg}$$

(8) You deposit $1600 in a bank account. Find the balance after 3 years if the account pays 4% annual interest yearly.

$$P(t) = 1600 \cdot (1.04)^3$$
$$= \boxed{\$\ 1{,}799.78}$$

(9) You buy a new computer for $2100. The computer decreases by 50% annually. When will the computer have a value of $600?

$$600 = 2100(1 - .5)^t$$
$$\frac{600}{2100} = \frac{2100(.5)^t}{2100}$$
$$\frac{2}{7} = (.5)^t$$

$$\log\left(\tfrac{2}{7}\right) = \log(.5)^t$$
$$\log\left(\tfrac{2}{7}\right) = t \cdot \log(.5)$$
$$\frac{\log\left(\tfrac{2}{7}\right)}{\log(.5)} = t$$
$$t \approx 1.81$$

$\boxed{2\ years}$

(10) You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. How long until you have 10mg of caffeine?

$$120(1 - .12)^t = 10$$
$$(.88)^t = \frac{10}{120}$$
$$\log(.88)^t = \log\left(\tfrac{1}{12}\right)$$

$$t \cdot \log(.88) = \log\left(\tfrac{1}{12}\right)$$
$$t = \frac{\log(1/12)}{\log(.88)}$$
$$t \approx 19.4\ hours$$

$\boxed{20\ hours}$

(11) The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. How long until the number of termites doubles?

$$1200(1.024)^t = 2400$$
$$(1.024)^t = 2$$
$$t \log(1.024) = \log 2$$

$$t = \frac{\log 2}{\log(1.024)}$$
$$t \approx 29.2\ days$$

$\boxed{30\ days}$