Review for Exam 1

Geometry, MTH 329 Spring 2026
Instructor: Abhijit Champanerkar

- Exam 1 will be held on **Monday March 2nd** and will be around an hour long.
- **Syllabus:** Chapter 3 and Spherical Geometry. See notes posted at:

  `https://www.math.csi.cuny.edu/~abhijit/329/resources/sg-notes.pdf`

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1. Straight edge and compass constructions:
   - **Basic constructions:** Copy segment, Add/subtract segments, Copy angle, Perpendicular bisector, Bisect angle, Equilateral triangle, given one side, Perpendicular line through given point off the line, Perpendicular line through given point on the line, Parallel line through a given point off the line.
   - **Constructions requiring several steps:** Copy a triangle, Regular hexagon, given one side, Square, given one side, Rectangle, given two sides, Rhombus, given two diagonals, Parallelogram, given two sides and one angle.
2. **Euclidean Geometry:**
   (a) Euclid's propositions: Justifying basic constructions, I.5, I.6, I.8 (SSS), I.17.
   (b) Given a line segment AB, a point C is equidistant from both A and B if and only if C lies on the perpendicular bisector of AB (Theorem 3.5 in book).
   (c) Of all line segments joining a point not on a given line to the line, the unique shortest segment is perpendicular to a given line (Theorem 3.11 in the book).
   (d) If the hypotenuse and a leg of right triangles are congruent then the the triangles are congruent (Theorem 3.12 in the book).
3. **Spherical Geometry:**
   - **Definitions & Examples:** Antipodal points, Lines on $\mathbb{S}^2$, equations and poles, Spherical distance, Angle between great circles, Spherical isometry, Antipodal map.

- **Prove the following statements** [1]
  - (a) If a point lies on a great circle, then its antipode also lies on it i.e if $P \in L_{\vec{n}}$ then $-P \in L_{\vec{n}}$. (Proposition 6)
  - (b) Any two distinct great circles intersect in a pair of antipodal points. (Proposition 8)
  - (c) Any two distinct non-antipodal points on $\mathbb{S}^2$ lie on a unique great circle. (Theorem 9)
  - (d) $|PQ|_{\mathbb{S}^2} \geq 0$, & $|PQ|_{\mathbb{S}^2} = 0$ if and only if $P = Q$ or $|PQ|_{\mathbb{S}^2} = |QP|_{\mathbb{S}^2}$. (Theorem 11)
  - (e) $|PQ|_{\mathbb{S}^2} = |QP|_{\mathbb{S}^2}$. (Theorem 11)
  - (f) Let P and Q be distinct points on $\mathbb{S}^2$. Then the set of points which are equidistant from P and Q is a great circle. (Theorem 18)
  - (g) Let P and Q be distinct points on $\mathbb{S}^2$. The equidistant great circle is the perpendicular bisector of the segment $\overline{PQ}$. (Theorem 18)
  - (h) Use Girard's theorem to prove that there are no squares on $\mathbb{S}^2$.
  - (i) Use Girard's theorem to explain why any map of $\mathbb{S}^2$ cannot preserve both angles and areas of regions on $\mathbb{S}^2$.
- **Computations**
  - (a) Find the great circle containing the points $P = (0, 1/2, \sqrt{3}/2)$ and $-P = (0, -1/2, -\sqrt{3}/2)$.
  - (b) Find the great circle containing the points. $P = (1/2, -1/2, 1/\sqrt{2})$ and $Q = (2/3, 1/3, -2/3)$.
  - (c) Find the distance between the points $P = (1/2, -1/2, 1/\sqrt{2})$, $Q = (2/3, 1/3, -2/3)$.
  - (d) Find the distance between the points $P = (0, 1/2, \sqrt{3}/2)$ and $-P = (0, -1/2, -\sqrt{3}/2)$.
  - (e) Find angles between the great circles $L_{\vec{j}}$ and $L_{-\vec{k}}$.
  - (f) Find angle between the great circles $L_{\langle 1/3, 2/3, 2/3 \rangle}$ and $L_{\langle -3/5, 4/5, 0 \rangle}$.
  - (g) Find the perpendicular bisector for the segments $\overline{PQ}$ where $P = (1/2, -1/2, 1/\sqrt{2})$, $Q = (2/3, 1/3, -2/3)$.
  - (h) Find the sides, angles and area of the triangle with vertices $P = (1, 0, 0)$, $Q = (0, -1, 0)$ and $R = (0, 0, -1)$.

[1] Reference numbers refer to the notes.