Homework 4

Geometry, MTH 329 Spring 2026
Instructor: Abhijit Champanerkar
Due: Monday April 27th [1] [2]

Reading

1. Read Section 10.5 and Eratosthenes (276–194 BCE) calculation of the circumference of the earth.
2. Read Section 11.5 - Constructing regular polygons.
3. Read Section 11.6 - About calculation of π.
4. Read AMS Feature Column **Euler's Polyhedral Formula** at `https://www.ams.org/publicoutreach/feature-column/fcarc-eulers-formula`

Problems

1. Using a straight edge and compass, and assuming a unit of 1 (e.g. 1 inch), construct the following:
   (a) $\sqrt{7}$.
   (b) Golden ratio $\phi = (1+\sqrt{5})/2$.
   (c) Regular pentagon.
   (d) Inscribe regular pentagon in a circle.
   (e) Construct regular 10-gon.
2. Find all n such that $3 \leq n \leq 100$ and the regular n-gon is constructible.
3. Given an equilateral triangle and a square, construct a regular 12-gon.
4. Given regular m-gon that is constructible and a regular n-gon that is constructible, prove that if m and n are relatively prime $(\gcd(m,n)=1)$, then a regular mn-gon is also constructible.

[1] Please upload pdf scan on Brightspace
[2] All section, chapter, page and example numbers refer to the book "Geometry: The Line and the circle" by Carroll and Rykken.

5. Assume that quadrilateral $ABCD$ is tangential with an inscribed circle of radius r. Prove that the area of $ABCD$ is given by $Area = r \cdot s$, where s is the **semiperimeter**,

$$s = \frac{AB + BC + CD + AD}{2}.$$

6. An **Archimedean solid** is a convex polyhedra whose faces are regular convex polygons of two or more different types arranged in the same way about each vertex with all sides the same length. There are 13 of them. See `https://en.wikipedia.org/wiki/Archimedean_solid`.
   Using an argument similar to the one we used to classify Platonic solids using Eulers polyhedral formula prove that:
   (a) There is only one Archimedaan solid which has an equal number of triangular and hexagonal faces.
   (b) There is only one Archimedaan solid which has square and hexagonal faces.

7. Find a polyhedron with holes for which Euler's polyhedral formula doe not hold.

8. Let P be a convex polyhedron with V vertices, E edges and F faces Prove that $E \leq 3V - 6$.

Hand-in: Problems 1e, 3, 5, 6a, 8