Homework 7

Introduction Mathematical Proof, MTH 301, Fall 2023
Instructor: Abhijit Champanerkar
Points: 30 **Due:** Monday Nov 20th, 2023

Reading: Its really important in this class that you read the book carefully, working through the Explorations and GYHD's as you go.

Sections 4.1, 4.2

Homework Problems (from text book):

1. Section 4.1, Pages 146-148: 1, 2, 7, 8, 13bc
2. Solve all the problems in the Functions Practice Problems Fun Pack (next page).

Handin Problems: These problems are to be handed in via Blackboard. Please see the instructions to how to submit hw. Write up clear solutions to the following problems.

Functions Practice Problem: 2bc, 3a, 4ac, 5d, 6b

For Problems 3, 4 and 5 below please note the following definition:
Let $f : X \to Y$ and let $B \subseteq Y$. "Inverse image of B" means

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}.$$

Please explore this definitions with a few examples.

Function Practice Problems Fun Pack

1. Suppose $f : X \to Y$. Prove that
   (a) If $A \subseteq B \subseteq X$, then $f(A) \subseteq f(B)$
   (b) If $C \subseteq D \subseteq Y$, then $f^{-1}(C) \subseteq f^{-1}(D)$.
   (c) For every $x \in X$, $f(\{x\}) = \{f(x)\}$.

2. Suppose $f : X \to Y$, and $A, B \subseteq X$. Prove that
   (a) $f(A \cup B) = f(A) \cup f(B)$
   (b) $f(A \cap B) \subseteq f(A) \cap f(B)$
   (c) Prove or disprove: $f(A) \cap f(B) \subseteq f(A \cap B)$

3. Suppose $f : X \to Y$, and $A, B \subseteq Y$. Prove that
   (a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
   (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
   (c) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$.

4. Suppose $f : X \to Y$ and $B \subseteq Y$. Prove that
   (a) $f[f^{-1}(B)] \subseteq B$.
   (b) Prove or disprove: $B \subseteq f[f^{-1}(B)]$
   (c) If f is onto, then $f[f^{-1}(B)] = B$

5. Suppose $f : X \to Y$. Prove that
   (a) For all $A \subseteq X$, $A \subseteq f^{-1}[f(A)]$.
   (b) Find a counterexample to the statement: For all $A \subseteq X$, $A = f^{-1}[f(A)]$

(c) If f is 1-1, then for all $A \subseteq X$, $A = f^{-1}[f(A)]$

(d) Suppose for all $A \subseteq X$, $A = f^{-1}[f(A)]$. Prove that f is 1-1.

6. Suppose $f : X \to Y$. Prove that

   (a) If f is 1-1, then for all $A, B \subseteq X$, $f(A \cap B) = f(A) \cap f(B)$.

   (b) Suppose that for all $A, B \subseteq X$, $f(A \cap B) = f(A) \cap f(B)$. Prove that f is 1-1.