Sample Problems for Exam 2

Calculus III, MTH 233, Fall 2017
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- Exam 2 will be held in class on Wednesday Nov 29.
- Review for Exam 2 will be held on Monday Nov 27.
- Best way to prepare for the midterm is to solve the Sample problems and Webwork Problems. You can now look solutions to Webwork problems!
- The exam will be shorter than Sample Problems sheet.

1. Find the cylindrical and spherical coordinates for the point \( P(\sqrt{3}, 1, 2\sqrt{3}) \).

2. Describe and sketch the surface given in spherical coordinates by \( \phi = \pi/3 \). Find the equations in cylindrical and cartesian coordinates for this surface.

3. Let \( f(x, y) = x^2y + y^2z \). If \( x = s + t, \ y = st, \ z = 2s - t \), compute \( \frac{\partial f}{\partial s} \) and \( \frac{\partial f}{\partial t} \).

4. Let \( f(x, y, z) = x + y^2z \) and \( x = 3s^2 + 2t, \ y = 3s - 2t^2 \) and \( z = s^2 - t^2 \). Compute \( \frac{\partial f}{\partial s}(2, -2) \) and \( \frac{\partial f}{\partial t}(2, -2) \).

5. Find all the critical points of \( f(x, y) = x^3 + y^2 - 3xy + 4 \), and classify them using the Second Derivative Test.

6. Find all the critical points of \( f(x, y) = 2y \cos x \) in the domain \([0, 2\pi] \times [-1, 1]\) and classify them using the second derivative test.

7. Find the critical points and analyze them using the Second Derivate Test for the following functions:
   (a) \( f(x, y) = x^2 + 2y^2 - 4xy + 6x \)  
   (b) \( f(x, y) = x^3 + 2y^3 - xy \)

8. Let \( f(x, y) = 2x^2 + y^2 - 4y + 3 \).
   (a) Find critical points of \( f \) on the region \( x^2 + y^2 < 9 \).
   (b) Find the exterme values on the boundary \( x^2 + y^2 = 9 \) using Lagrange Multipliers.
   (c) Find the exterme values of \( f \) on \( x^2 + y^2 \leq 9 \) using the above information.

9. Use Lagrange multipliers to find the minimum and maximum value of the given function subject to the given restraint:
   (a) \( f(x, y) = 3x - 2y \) on the circle \( x^2 + y^2 = 4 \).
   (b) \( f(x, y) = x^2y \) on the ellipse \( 4x^2 + 9y^2 = 36 \).

10. Use Lagrange multipliers to find the dimensions of a cylindrical can of fixed volume \( V = 1 \) with minimal surface area, including the top and bottom of the can.
11. A line with equation \(\frac{x}{a} + \frac{y}{b} = 1\), with \(a, b > 0\), together with the coordinate axes forms a triangle with area \(A = \frac{ab}{2}\). Using Lagrange multipliers find the line that minimizes the area \(A\), if the line is constrained to pass through the point \(P = (1, 2)\).

12. Evaluate the following double integrals.
   (a) \(\int_{D} xy + 2x + 3y \, dA\) where \(D\) is the region in the first quadrant bounded by \(x = 1 - y^2\), \(x = 0\), \(y = 0\).
   (b) \(\int_{D} xe^y \, dA\) where \(D\) is bounded by \(x = 1\), \(y = 0\), \(y = x^2\).
   (c) \(\int_{D} xy \, dA\) where \(D\) is bounded by \(y = 5 - x^2\), \(y = x^2 - 3\).

13. Evaluate the following double integrals using polar coordinates.
   (a) \(\int_{D} (x^2 + y^2)^{3/2} \, dA\) where \(D\) is bounded by \(y = 0\), \(y = \sqrt{3}x\), \(x^2 + y^2 = 9\).
   (b) \(\int_{0}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x^2 + y^2 \, dy \, dx\)

14. Change the order of integral and evaluate.
   (a) \(\int_{0}^{1} \int_{\sqrt{y/2}}^{\sqrt{y}} e^{-x^2} \, dx \, dy\)
   (b) \(\int_{0}^{\pi/2} \int_{0}^{2} \sin(x^4) \, dx \, dy\)

15. Evaluate the following triple integrals
   (a) \(\iiint_{E} x^2 \, dV\) where \(E\) is \(\{(x, y, z) | 0 \leq x \leq 2, 0 \leq y \leq 2x, 0 \leq z \leq x\}\).
   (b) \(\iiint_{T} y \, dV\) where \(T\) is the tetrahedron bounded by the planes \(x = 0\), \(y = 0\), \(z = 0\) and \(2x + y + z = 2\).

16. Evaluate the following integrals using Cylindrical or Spherical Coordinates.
   (a) \(\iiint_{H} z^5 \sqrt{x^2 + y^2 + z^2} \, dV\) where \(H\) is the solid hemisphere with the center at the origin and radius 1 which lies above the \(xy\)-plane.
   (b) Evaluate \(\iiint_{E} \sqrt{x^2 + y^2} \, dV\) where \(E\) is the solid bounded by the paraboloids \(z = 2 - x^2 - y^2\) and \(z = x^2 + y^2 - 2\).
   (c) \(\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{x^2 + y^2 + z^2}} \, dz \, dy \, dx\)
   (d) \(\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{1+\sqrt{1-x^2-y^2}}^{1-\sqrt{1-x^2-y^2}} 2 \, dz \, dy \, dx\)

17. Set up the integrals for volumes of the given solids and indicate with coordinates you will use. Do not evaluate the integrals.
   (a) The volume of the tetrahedron bounded by the plane \(3x + 2y + z = 6\) in the first octant.
(b) The volume of ice-cream bounded by the cone \( z = \sqrt{x^2 + y^2} \) and the sphere \( x^2 + y^2 + z^2 = 1 \).

(c) The volume inside the cylinder \( x^2 + y^2 = 4 \) and the ellipsoid \( 4x^2 + 4y^2 + z^2 = 64 \). the planes \( z = 0, \ y = 0, \ y = x, \ z = 1 \) in the first octant.

(d) The volume of a wedge of cheese bounded by the cylinder \( x^2 + y^2 = 1 \), and the planes \( z = 0, \ z = 1, \ y = 0, \ y = x \).

(e) The volume of the region between the paraboloids \( z = x^2 + y^2 - 1 \) and \( z = 1 - x^2 - y^2 \).

18. Find the curl and divergence of the following vector fields.

(a) \( \mathbf{F}(x, y, z) = (yz, xz, xy + z^2) \)

(b) \( \mathbf{F}(x, y, z) = (z - y^2, x + z^3, y + x^2) \)

(c) \( \mathbf{F}(x, y, z) = (y/x, y/z, z/x) \)

(d) \( \mathbf{F}(x, y, z) = (e^y, \sin x, z \cos x) \)

19. Show that (a) \( \text{curl}(\nabla f) = 0 \) (b) \( \text{div}(	ext{curl}(\mathbf{F})) = 0 \).