

# Solutions to Sample Problems for Exam 1

Calculus I, MTH 231  
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1.

a.  $y = 5x/2 - 23/2$ ,  $(2,1)$  not on line.    b.  $y = 2x$

2.

$f \circ g(x) = \ln(x^2 - 9)$ ,  $g \circ f(x) = (\ln x)^2 - 9$ ,  $f \circ f(x) = \ln(\ln x)$ ,  $g \circ g(x) = (x^2 - 9)^2 - 9$

3.

a. Factor num & den, -3    b. Rationalize,  $1/6$     c. Expand, 8    d.  $15/2$   
g.  $9/5$     h. Rationalize,  $1/6$     i.  $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ ,  $7/2$     j. 0

4.

a.  $3/5$     b. 0    c.  $\infty$     d.  $-\sqrt{2}/3$

5.

$\frac{|x+1|}{x+1} = \begin{cases} 1 & x > -1 \\ -1 & x < -1 \end{cases}$ , right hand limit = 1  $\neq$  -1 = left-hand limit

6.

$f$  cts at  $x=2 \Rightarrow \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x) \Rightarrow \lim_{x \rightarrow 2^+} x-4 = \lim_{x \rightarrow 2^-} ax^2+1$   
 $\Rightarrow -2 = 4a+1 \Rightarrow a = -3/4$

7.

i)  ii)  iii)  DNE iv)  v)  vi)   
 $f$  discts at  $x=0$  &  $x=3$  (undefined at  $x=3$ )

8.

a)  $1, 1, \infty, \infty, 3, 0.75$   
b)  $f$  discts at  $x=-2$  removable discty,  $x=-1$  jump discty  
 $x=1$  infinite discty

9.

a)  $-\infty, \infty, \infty, -\infty, 2, 2$   
Vert asympt:  $x=3, x=-3$  Hor. asympt:  $y=2$

10.

Use  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$a) f(x+h) - f(x) = 2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1) = 2(x^2 + 2xh + h^2) + 3(x+h) + 1 - (2x^2 + 3x + 1) \\ = 4xh + 4h^2 + 3h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(4xh + 4h^2 + 3h)}{h} = 4x + 3$$

$$b) f(x+h) - f(x) = \frac{2}{xh+1} - \frac{2}{x+1} = \frac{2(x+1) - 2(x+h)}{(x+h+1)(x+1)} = \frac{-2h}{(x+h+1)(x+1)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{-2h}{(x+h+1)(x+1)} = \frac{-2}{(x+1)^2}$$

$$c) f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \text{ rationalize} = \lim_{h \rightarrow 0} \frac{(x+h+3) - (x+3)}{(\sqrt{x+h+3} + \sqrt{x+3})h} = \frac{1}{2\sqrt{x+3}}$$

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$$y' = e^x, y = \frac{1}{4}, \text{ slope} = \frac{1}{4}, e^x = \frac{1}{4}, x = \ln \frac{1}{4}, \text{ pt} = (\ln \frac{1}{4}, \frac{1}{4})$$

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Solutions to Problems 11, 12, 13, 15 on next page.

$$11. a. y' = 3x^2 + 3 + \frac{1}{3x^{2/3}}$$

$$b. y = \frac{x^5}{\sqrt{x}} + \frac{4}{\sqrt{x}} = x^{9/2} + 4x^{-1/2}$$

$$y' = \frac{9}{2}x^{7/2} - \frac{2}{x^{3/2}}$$

$$c. y = x^2 - 3 + \frac{5}{x^2}$$

$$y' = 2x - \frac{10}{x^3}$$

$$d. y' = e^x + 2x$$

$$12 a. y' = 1 - \frac{1}{2\sqrt{x}}, \quad y'|_{x=1} = \frac{1}{2}$$

$$y - 2 = \frac{1}{2}(x - 1), \quad \boxed{y = \frac{x}{2} + \frac{3}{2}}$$

$$b. y = x^2 + 2x - 15$$

$$y' = 2x + 2, \quad y'|_{x=2} = 6$$

$$y - (-7) = 6(x - 2)$$

$$\boxed{y = 6x - 5}$$

$$13 a. y' = 3x^2 - 12x = 0$$

$$3x(x-4) = 0 \quad x = 0, 4$$

$$b. y = 3x^3 + 12x^2 - 20x.$$

$$y' = 9x^2 + 24x - 20 = 0$$

$$9x^2 - 6x + 30x - 20 = 0$$

$$\cancel{9x^2} (3x+10)(3x-2) = 0$$

$$x = -10/3, 2/3$$

$$15. a) \neq f'(a) \quad b) \neq f'(a)$$

$$c) f(t) = t^{500} \\ a = 1$$

$$d) f(x) = 3^x \\ a = 2$$