Summary of Derivative tests and curve sketching

Calculus I, MTH 231
Instructor: Abhijit Champanerkar
Topic: Sections 4.3-4.4-4.5

1. Increasing/Decreasing Test Let f be differentiable on (a,b). Then

- $f'(x) > 0$ on $(a,b) \implies f$ is increasing on (a,b).
- $f'(x) < 0$ on $(a,b) \implies f$ is decreasing on (a,b).

2. First derivative Test for critical points Let f be differentiable and let c be a critical point of $f(x)$. Then

- $f'(x)$ changes from $+$ to $-$ at $c \implies f(c)$ local maximum.
- $f'(x)$ changes from $-$ to $+$ at $c \implies f(c)$ local minimum.

To find Monotonicity Compute $f'(x) \rightarrow$ Solve $f'(x) = 0$ to get critical points $\rightarrow$ Find intervals of increase/decrease using **Increasing/Decreasing Test** $\rightarrow$ Analyse critical points using **First derivative Test**.

3. Concavitiy Test Assume $f''(x)$ exists on (a,b). Then

- $f''(x) > 0$ on $(a,b) \implies f$ is concave up (CU) on (a,b).
- $f''(x) < 0$ on $(a,b) \implies f$ is concave down (CD) on (a,b).

4. Inflection point Test Assume $f''(c)$ exists. Then

- $f''(c) = 0$ and $f''(x)$ changes sign at $c \implies f(x)$ has an inflection point at $x = c$.

To find Concavity Compute $f''(x) \rightarrow$ Solve $f''(x) = 0 \rightarrow$ Find intervals of concavity using **Concavity Test** $\rightarrow$ Find inflection points using **Inflection point Test**.

5. Second derivative Test for critical points Let c be a critical point of $f(x)$. If $f''(c)$ exists, then

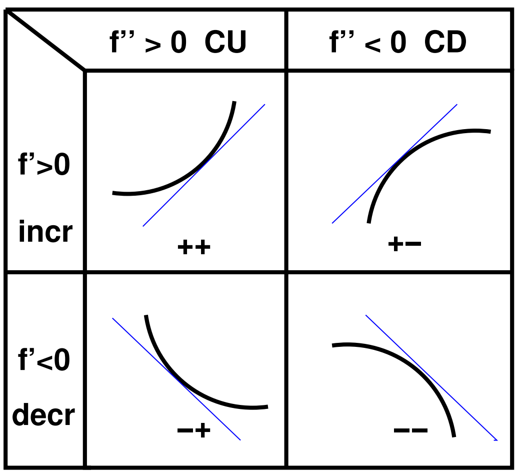
- $f''(c) > 0 \implies f(c)$ is local minimum.
- $f''(c) < 0 \implies f(c)$ is local maximum.
- $f''(c) = 0 \implies$ inconclusive, use First derivative test.

Curve sketching

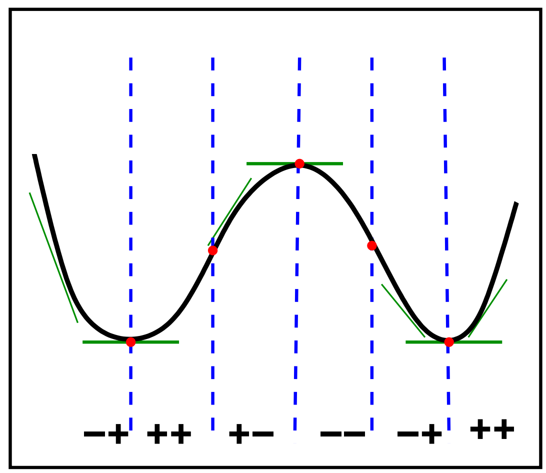
A **transition point** is a point in the domain of f at which either f' changes sign (local min or max) or f'' changes sign (point of inflection).

Steps in curve sketching:

- **Step 1:** Determine signs of f' and f''.
- **Step 2:** Note transition points and sign combinations of f' anf f''.
- **Step 3:** Determine asymptotes of f.
- **Step 4:** Draw arcs of appropriate shape and asympototes.



(a)



(b)

Figure 1: (a) The four basic shapes (b) Graph of a function with transition points .