Review for Final

Calculus I Computer Lab, MTH 229, Fall 2021

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1. **Final Exam** will be held on Monday Dec 20th 2:30 - 4:25, 1S-108, in-person.
2. Read the class notebooks we used during class (from the folder https://github.com/mth229/229-projects on gesis or mybinder), the Webwork projects and review problems for the exam.
3. Here is a short review of some of the Julia commands we have used this semester.

Julia commands.

1. (a) Help on commands: `?commandname`
   (b) Use commands `using MTH229, using SimplePlots`
   (c) Use Julia notebook version 1.5.3
2. Order of operations in Julia.
3. In-built functions: `sqrt, cbrt, sin, cos etc, sind, cosd etc, asin, acos exp, log, log(b,x)`
4. Functions defined using ternary operator:
   `ternary operator predicate ? expression1 : expression2`
   e.g. `f(x) = x <= 1 ? 35 : 35 + 10 * (x - 1)`
5. Graphing functions: `plot` and `plot!` .
6. Arrays and Lists: `range, map, plot, scatter,`
   For e.g. `xs = range(0, 10, length=50); ys = f.(xs); ys = map(sin, xs), plot(xs, ys)`
7. Finding Zeros: `roots(f)` (for zeros of polynomials only!), `bisection(f,a,b)` (uses bisection method), `fzero(f,a)` (finds zero near a for **any** function with high accuracy), `fzeros(f,a,b)` (finds all zeros in the interval (a, b) for **any** function, not much accuracy).
8. Limits: `lim(f,c)` (approximate the limit of a function).
9. Derivatives: `secant(f,a,b), tangent(f,a)` (secant and tangent line), `f'(a), f''(a), f'''(a)` (numerically)
10. Newton's method: `newton(f, a)`
11. First and second derivatives and Extrema: `plotif(f,f',a,b)` (plots increasing and decreasing parts), `plotif(f,f'',a,b)` (plots concave up and down parts) `fzero(f, a)`
12. Integration: `integrate(f, a, b)` (find definite integral), `riemann(f, a, b, n, method="type")` (find Riemann sums using different methods), `quadgk(f,a,b)` (uses Gauss quadrature method, also returns error)

13. Symbolic commands:

Limits

```
f(x) = sin(x)/x
@syms x
limit(f(x), x => 0)
```

Derivatives

```
f(x) = exp(x^2)
@syms x
diff(f(x), x)
```

Integration

```
f(x) = x^2
@syms x
integrate(f(x), x)
integrate(f(x), (x, 0, 1))
```

Math 229 Final Exam Review

Problem 1. Convert the following Julia expressions to standard mathematical expressions. Use parentheses if necessary to clearly indicate the order of operations:

a. `b-a/b-b/c`

b. `sin(1/4*x^2)/2x^3`

Problem 2. Write out the Julia commands for the following mathematical expressions.

a. $f(x) = \dfrac{\sin^2(4x)}{\sqrt{2x}+2}$

b. $g(x) = \dfrac{\tan^{-1}(3x)}{e^{2x}-1}$

Problem 3. Let $f(x) = \tan(x/6)\cdot\cos(x+3)$, for $0 \le x \le 2\pi$.
Find ALL points a such that $f(a) = 0$, rounded to four (4) decimal places.

Problem 4. Write the Julia commands for this function: $h(x) = \begin{cases} 4 - \frac{7}{x^2} & x \le -1 \\ 3 - 1/x & x > -1 \end{cases}$

Compute the following values. Truncate answers to 4 decimal places.

$h(h(h(\sqrt{6}))) =$ ________________ $h(h(h(-\sqrt{6}))) =$ ________________

Problem 5. Plot the following functions on the interval $(\pi, 5)$.

$$f(x) = \frac{\sin(12x)}{e^x} \qquad g(x) = \frac{\cos(12x)}{x^3}$$

a. How many times do the two curves intersect for $\pi < x < 5$? __________

b. What is the number of local maxima (peaks) for each function? (Exclude endpoints)
Number of local maxima for $f(x)$ is __________.
Number of local maxima for $g(x)$ is __________.

Problem 6. Find the minimum point (x–value) for $0 < x < \pi$ for

$$h(x) = \left(\cos(x) + \frac{1.7}{(x-\pi)^2}\right)$$

a. Exact minimum x–value to three (3) decimal places:__________.

b. Write the precise Julia commands you used to solve this problem.

Problem 7. Let $g(x) = x^5 - 4x + 2$.
What is the SMALLEST real root, rounded to four (4) decimal places?
Hint: If $a < b$, then a is smaller than b, so -5 is smaller than -2.

Problem 8. Let $h(x) = e^{2x} - 4e^x + 4$.

a. Use `fzeros` to find all roots of $h(x)$, rounded to four (4) decimal places.

b. Use `fzero(h,-10,10)` to find all roots of $h(x)$. Explain what goes wrong.
Hint: Graph the function.

Problem 9. Compute the following limits. Round answers to 4 decimal places.

a. $\displaystyle\lim_{x\to 5} \frac{\cos(2x^3 + \pi/2)\sin(x-5)}{x-5}$ =________________________

b. $\displaystyle\lim_{x\to 3} \frac{\log((x-3)^4 + 2x - 5)}{x-3}$ =________________________

c. $\displaystyle\lim_{x\to 0} (\cos(x))^{(3/x^2)}$ =________________________

Problem 10. Compute the EXACT answer (symbolically) for the following limits.

a. $\displaystyle\lim_{x\to 0} (\cos(x))^{(3/x^2)}$ =________________________

b. $\displaystyle\lim_{x\to 0^+} \sqrt{\frac{5}{x}}\,\sin\left(\frac{\sqrt{x}}{3}\right)$ =________________________

Problem 11. $f(x) = \tan(x/6)\cdot\cos(x-0.8)$, for $0 \le x \le 2\pi$.

Find the x-coordinates in this interval for the following points, accurate to 4 decimal places.

a. Points where $f'(x) = 0$:

b. Points where $f(x) = f'(x)$:

c. Points where $f''(x) = x^3$:

Problem 12. Use Newton's Method to find all zeros of $f(x) = \sqrt{x+1}\cos(x)$
for $0 \le x \le 10$.

a. Graph the function on this interval.

b. Write all the necessary Julia commands to use Newton's Method.

c. Write the answers, accurate to 4 decimal places.

Problem 13. Let $f(x) = e^{(x/3)} \sin(x+1)$ for $0 \le x \le 10$.
Write both the Julia commands and your answers, accurate to 4 decimal places.

a. Find all critical points of $f(x)$; i.e. points where $f'(x) = 0$.

b. Find all inflection points of $f(x)$; i.e. points where $f''(x) = 0$.

Problem 14. Write the answers, accurate to 4 decimal places.

$$f(x) = \sqrt{x+2}\cos(x+1) + x^3 \sin(2x) \quad \text{for} \quad 5 \le x \le 10.$$

a. Find all critical points of $f(x)$.

b. Find all inflection points of $f(x)$.

c. Where is $f(x)$ decreasing? Use interval notation.

d. Where is $f(x)$ concave down? Use interval notation.

e. Classify the critical points as max, min or neither <u>using the first derivative test</u>.

f. Classify the critical points as max, min or neither <u>using the second derivative test</u>.

Problem 15. Let $f(x) = \dfrac{x+1}{\sqrt{x^3+2}}$.

We want to compute $\displaystyle\int_3^7 f(x)\,dx$. But the command `integrate` will not work here.

a. Approximate $\displaystyle\int_3^7 f(x)\,dx$ by Riemann sums with these methods and values of n:

	Left endpoint method	Right endpoint method	Simpsons method	Trapezoid method
$n = 100$				

b. What is the answer using the command `quadgk` ?

c. How accurate is the answer given by `quadgk` ? i.e., what is the max error?

d. Which is the best answer in the table above?