SOLUTIONS TO Sample Problems for Exam 3

(1a) $\angle C = 180-(50+68) = 62^\circ$

$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow a = \frac{\sin 50}{\sin 62} \times 230 = 201.28$

$\frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow b = \frac{\sin 68}{\sin 62} \times 230 = 243.62$

(1b) $\angle A = 180-(100+10) = 70^\circ$

$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow a = \frac{\sin 70^\circ}{\sin 100} \times 115 = 109.73$

Similarly. $b = \frac{\sin 10}{\sin 100} \times 115 = 20.28$.

(1c) $\sin B = \frac{\sin A}{a} \times b = \frac{\sin 110}{28} \times 15 = 0.50$

$B_1 = 30^\circ$, $C_1 = 180-(110+30) = 40^\circ$, $c = \frac{\sin 40}{\sin 110} \times 28 = 19.15$

$B_2 = 180-30 = 150^\circ$, $C_2 = 180-(110+150)$

$C_2 = -80 \rightarrow\leftarrow$

(2a) $x^2 = 5^2+6^2-2\times5\times6\cos 100 = 71.41$, $x = 8.45$

(2b) $x^2 = 100^2+210^2-2\times100\times210\cos 40 = 21926.13$ | $x = 4.81$

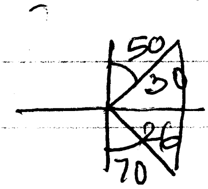
(2c) $x^2 = 100+36-2\times100\times36\cos 30^\circ$

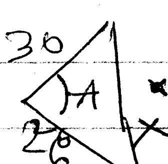
$x = 78.10$

(3) Use Heron's formula. $\text{Area} = \sqrt{8\times9\times6\times3} = 36$

$s = 9+12+15/2 = 18$

(4)





$A = 60$ ~~In one hour~~

$x^2 = 30^2+26^2-2\times30\times35\cos 60$

$= 796$, $x = 2\sqrt{199} = 28.21$

5a) $LHS = \dfrac{\cos x}{1/\cos x} + \dfrac{\sin x}{1/\sin x} = \cos^2 x + \sin^2 x = 1 = RHS$

5b) $LHS = \left(\dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}\right)(\cos x + 1) = \dfrac{(\cos x - 1)(\cos x + 1)}{\sin x} = \dfrac{\cos^2 x - 1}{\sin x}$

$= \dfrac{-\sin^2 x}{\sin x} = -\sin x$
$= RHS.$

5c) $LHS = \dfrac{\sin^2 u}{\cos^2 u} - \sin^2 u = \dfrac{\sin^2 u - \sin^2 u \cos^2 u}{\cos^2 u}$

$= \dfrac{\sin^2 u(1 - \cos^2 u)}{\cos^2 u} = \tan^2 u \sin^2 u = RHS.$

5d) LHS
$\cos 2t = \underset{\text{sum formula}}{\cos(t+t)} = \cos t \cos t - \sin t \sin t$
$= \cos^2 t - \sin^2 t.$
$= RHS$

5e) $RHS = \left(\dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}\right)^2 = \dfrac{(1+\sin x)^2}{\cos^2 x} = \dfrac{(1+\sin x)^2}{1 - \sin^2 x}$

$= \dfrac{(1+\sin x)^{\cancel{2}}}{(\cancel{1+\sin x})(1-\sin x)} = \cancel{RHS}\ LHS.$

6a) $\tan 15° = \tan \dfrac{30°}{2} = \dfrac{\sin 30°}{1 + \cos 30°} = \dfrac{1/2}{1 + \sqrt{3}/2} = \dfrac{1}{2+\sqrt{3}}$

6b) $\cos 195° = \cos(60 + 135) = \tfrac{1}{2}\cdot\tfrac{-\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{3}}{2} = \dfrac{-(\sqrt{2} + \sqrt{2}\sqrt{3})}{4}$

$\cos 135 = \cos(90+45) = -\cos 45 = -\sqrt{2}/2, \quad \sin 135 = \sqrt{2}/2$

6c) $\sin 22.5 = \sin(45/2) = \sqrt{\dfrac{1 - \cos 45}{2}} = \sqrt{\dfrac{1 - \sqrt{2}/2}{2}} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

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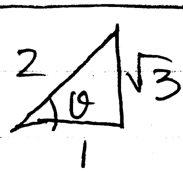
7. 

$\cos\theta = \dfrac{\sqrt{3}}{2} \quad \sin\theta = 1/2 \quad \theta = 30°.$

$2\left(\tfrac{1}{2}\cos x - \tfrac{\sqrt{3}}{2}\sin x\right) = 2(\sin 30 \cos x - \cos 30 \sin x)$
$= 2\sin(30 - x)$

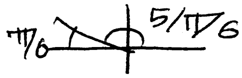
8. $\cos x = -4/5 \quad 180° < x < 270° \Rightarrow 90° < x/2 < 135°$
$x/2$ Quad II.

$\sin x/2 = \sqrt{\dfrac{1 - \cos x}{2}} = \sqrt{\dfrac{1-(-4/5)}{2}} = \sqrt{9/10} = \dfrac{3}{\sqrt{10}}$

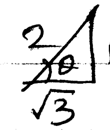
$\cos x/2 = -\sqrt{\dfrac{1 + \cos x}{2}} = -\sqrt{\dfrac{1 - 4/5}{2}} = -\sqrt{\dfrac{1}{10}}$

$\tan x/2 = -\sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$
$= -\sqrt{9/1} = -3$

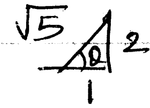
(3)

(9a) 

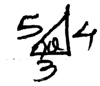
$\sin^{-1}(\sin 5\pi/6) = \sin^{-1}(\sin \pi/6) = \pi/6$.
($\sin^{-1}$ takes values $-\pi/2$ to $\pi/2$)

(9b) $\tan(\sin^{-1} 1/2)$, $\theta = \sin^{-1} 1/2$ $\sin\theta = 1/2$ 

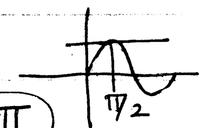
$\tan\theta = 1/\sqrt{3}$.

(9c) $\theta = \tan^{-1} 2$, $\tan\theta = 2$ 

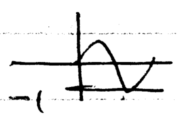
, $\cos\theta = 1/\sqrt{5}$

(9d) $\theta = \sin^{-1} 4/5$, $\sin\theta = 4/5$ 

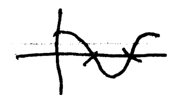
$\tan\theta = 4/3$

(10a) $\sin^2 x = 1$, $\sin x = \pm 1$ $\sin x = 1$ 

or $x = \pi/2 + 2n\pi$



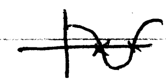
$\sin x = -1$, $x = 3\pi/2 + 2n\pi$.

(10b) ~~$\cos^2 x = 1/2$~~ $\cos 2x = 2\cos^2 x - 1 = 0$ 

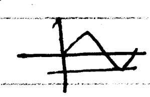
$2x = \pi/2 + 2n\pi$ or $3\pi/2 + 2n\pi$.

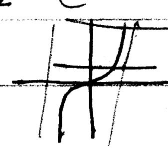
$x = \pi/4 + n\pi$ or $3\pi/4 + n\pi$.

(10c) $\cos x(\sin x - 2) = 0$ $\cos x = 0$, $\sin x = 2$
no solns.

 $x = \pi/2 + 2n\pi$ or $3\pi/2 + 2n\pi$.

(10d) $A = \sin x$, $A^2 - 2A - 3 = 0$ $(A-3)(A+1) = 0$.
$A = 3, -1$, $\sin x = 3$ no solns

$\sin x = -1$  $x = 3\pi/2 + 2n\pi$.

(10e) $\tan 3x = 1$  $3x = \pi/4 + n\pi$, $x = \pi/12 + n\pi/3$