SOLUTIONS TO SAMPLE PROBLEMS TO EXAM 1
MTH 130, Spring 2014

1\. a) $x^2 - 2x - 15 \geq 0$

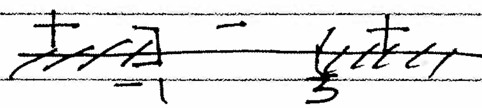
$(x-5)(x+3) \geq 0$

zeros $x = 5, -3$

Domain: $(-\infty, -3] \cup [5, \infty)$

b) $\dfrac{x+1}{x-3} \geq 0$ zeros: $x = -1$

VA: $x = 3$ (vertical asymptote)



Domain: $(-\infty, -1] \cup (3, \infty)$

2\. a) $f+g\,(x) = \sqrt{x+3} + \dfrac{1}{2-x^2}$

Domain: $\{x \,/\, x \geq -3,\ x \neq \pm\sqrt{2}\}$

domain(f) domain(g).

b) $(f+g)(1) = 2 + \dfrac{1}{2-1} = 3$

$(f-g)(1) = 2 - \dfrac{1}{2-1} = 1$

$fg\,(0) = \sqrt{3} \times \dfrac{1}{2}$, $\dfrac{f}{g}(1) = \dfrac{\sqrt{4}}{1/(2-1)} = 2 \times 1 = 2$

c) $f \circ g(x) = f(g(x)) = \sqrt{\dfrac{1}{2-x^2} + 3}$

$g \circ f(x) = g(f(x)) = \dfrac{1}{2-(x+3)}$

3. $a+2b = 1+2i + 2(3-i) = 7$

$4b = (1+2i)(3-i) = 3+2+i(6-1) = 5+5i$

$\frac{a}{b} = \frac{(1+2i)}{(3-i)}\frac{(3+i)}{(3+i)} = \frac{3-2+i(6+1)}{9+1} = \frac{1+7i}{10}$

$\frac{b}{a} = \frac{3-i}{(1+2i)}\frac{(1-2i)}{(1-2i)} = \frac{3-2+i(-1-6)}{1+4} = \frac{1-7i}{5}$

4. a). $3-i$ — $3-i$, $3+i$ roots } sum $= 6 = -b$, product $= 10 = c$ $\quad x^2-6x+10$

$4i$ — $4i$, $-4i$ roots } sum $= 0 = b$, product $= 16$ $\quad x^2+16$

$p(x) = (x^2+16)(x^2-6x+10)$

b). $2 \longrightarrow (x-2)$

$-3 \longrightarrow (x+3)$

$2+3i$ — $2+3i$, $2-3i$ roots } sum $= 4$, product $= 4+9 = 13$ $\quad x^2-4x+13$

$1-2i$ — $1-2i$, $1+2i$ roots } sum $= 2$, product $= 1+4 = 5$ $\quad x^2-2x+5$

$p(x) = (x-2)(x+3)(x^2-4x+13)(x^2-2x+5)$

In general if $\alpha+\beta i$ root then $\alpha-\beta i$ root & quadratic factor x^2+bx+c then $-b = 2\alpha = \underbrace{(\alpha+\beta i)+(\alpha-\beta i)}_{=\text{sum}}$, $c = \alpha^2+\beta^2 = \text{product}$

5. a) $f(x) = x^2(x^4+7x^2-18)$
$= x^2(x^2+9)(x^2-2)$

$x^2+9=0 \Rightarrow x^2=-9, \; x=\pm\sqrt{-9}=\pm 3i$
$x^2-2=0 \Rightarrow x^2=2, \; x=\pm\sqrt{2}$

$f(x) = x^2(x+3i)(x-3i)(x+\sqrt{2})(x-\sqrt{2})$

b) $f(x) = x^3+16x = x(x^2+16)$

$x^2+16=0 \Rightarrow x^2=-16, \; x=\pm\sqrt{-16}=\pm 4i$
$f(x) = x(x+4i)(x-4i)$

6. a) $f(x) = x^2+4x+5 = (x+2)^2+1$
$h=-2, \; K=1$
min value $= K = 1$

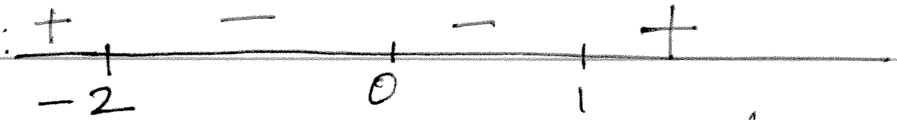
b) $f(x) = -2x^2+4x+4$
$h = -b/2a = \frac{-4}{-4} = 1, \; K = f(1) = 6$

$f(x) = -2(x-1)^2+6$

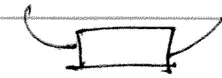
max $= K = 6$

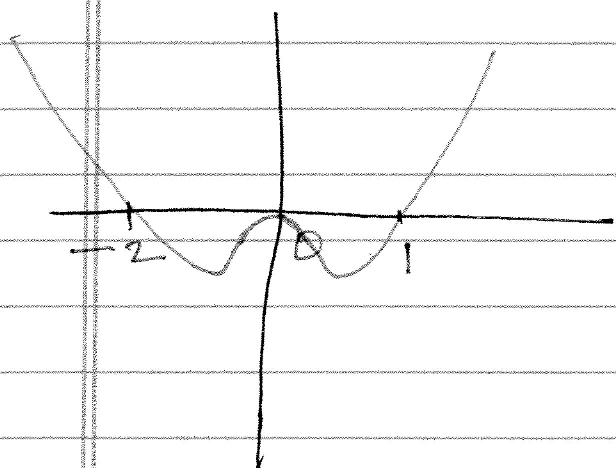
7a. zeros: $x = 0, 1, -2$

signs:



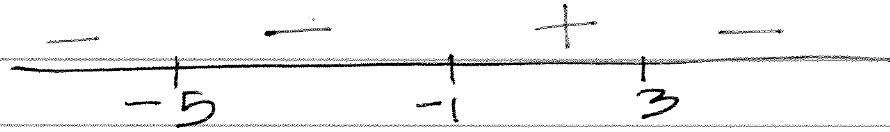
end behaviour: $x^2 \cdot x \cdot x = x^4$



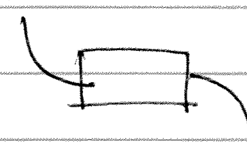


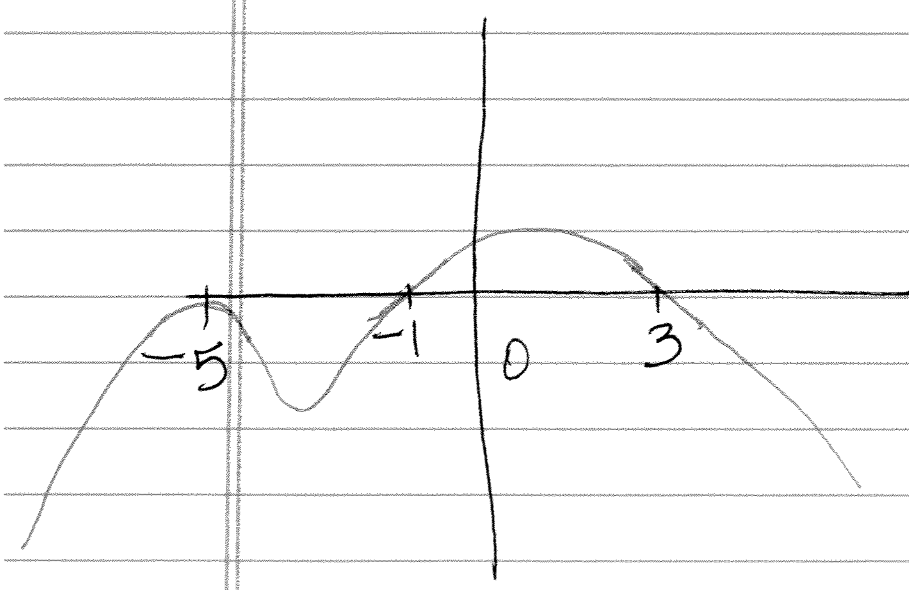
b: zeros: $x = -1, 3, -5$.

signs:



end behaviour: leading term $= -x^3 \cdot x \cdot x = -x^5$





8a $x^2-7x+12 = (x-4)(x-3)=0$

zeros $= x=4,3$

$+ \quad - \quad +$

$3 \quad 4$

≥ 0 on $(-\infty,3] \cup [4,\infty)$

b. $x-x^3 = x(1-x^2) = x(1-x)(1+x)$

zeros: $x=0,1,-1$

$+ \quad - \quad + \quad -$

$-1 \quad 0 \quad 1$

<0 on $(-1,0) \cup (1,\infty)$

c. $x^2+3x-18 = (x+6)(x-3)$ $\qquad x^2+3x \leq 18$

zeros: $x=3,-6$ $\qquad x^2+3x-18 \leq 0$

$+ \quad - \quad +$

$-6 \quad 3$

≤ 0 on $[-6,3]$

d. $\dfrac{x+1}{x-4}-2>0$ $\qquad \dfrac{9-x}{x-4}>0$

zeros: $x=9$, vert. asympt: $x=4$

$- \quad + \quad -$

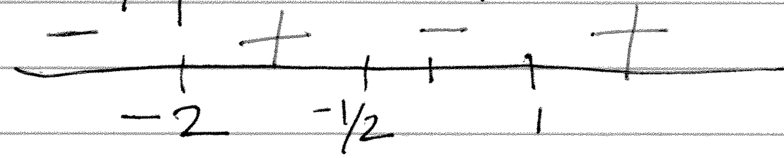
$4 \quad 9$

>0 on $(4,9)$

8e. $\dfrac{x-1+x+2}{(x-1)(x+2)} = \dfrac{2x+1}{(x-1)(x+2)} \le 0$

zeros: $x = -1/2$

Vert. asymp VA: $x = 1, -2$



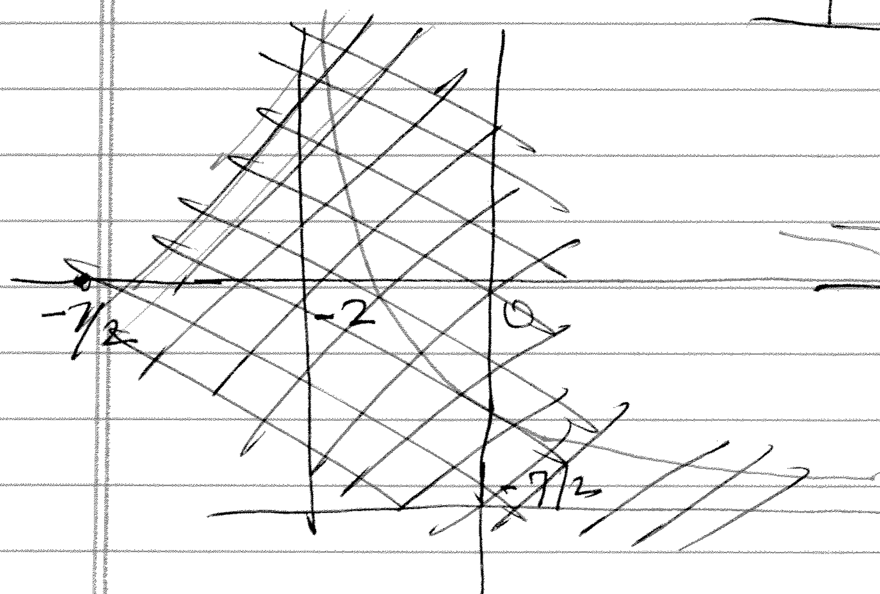
≤ 0 on $(-\infty, -2) \cup [-1/2, 1)$

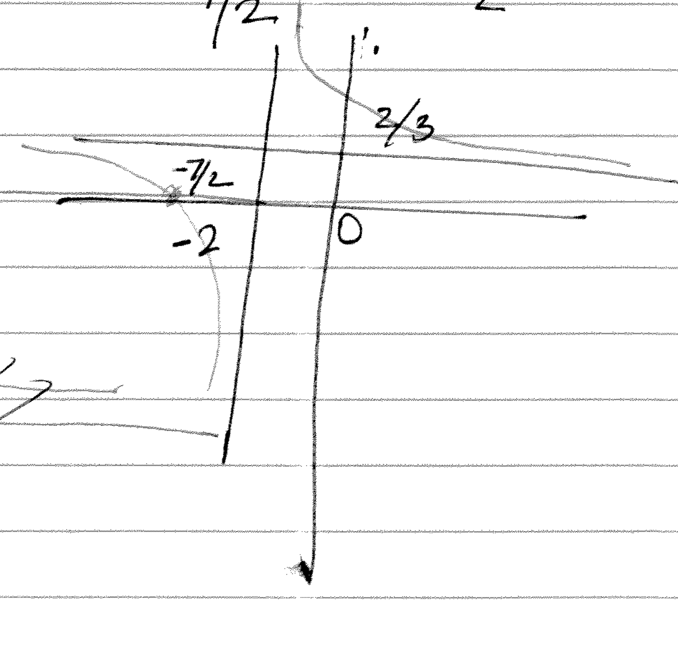
9. a(i) domain: $\{x \mid x \neq -2\}$

VA: $x = -2$ HA: $\dfrac{2x}{3x} = 2/3$

zeros: $x = -7/2$

signs:





$$\frac{x+4}{x^2+3x-10} = \frac{x+4}{(x+5)(x-2)}$$

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9(ii) domain: $\{x \mid x \neq 2, -5\}$.

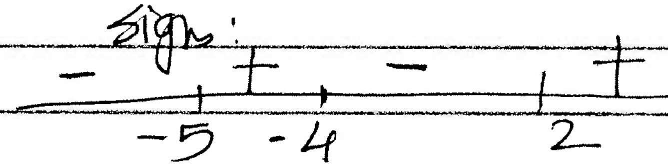
HA: $\frac{x}{x^2} = \frac{1}{x} \to 0$ as $x \to \pm\infty$

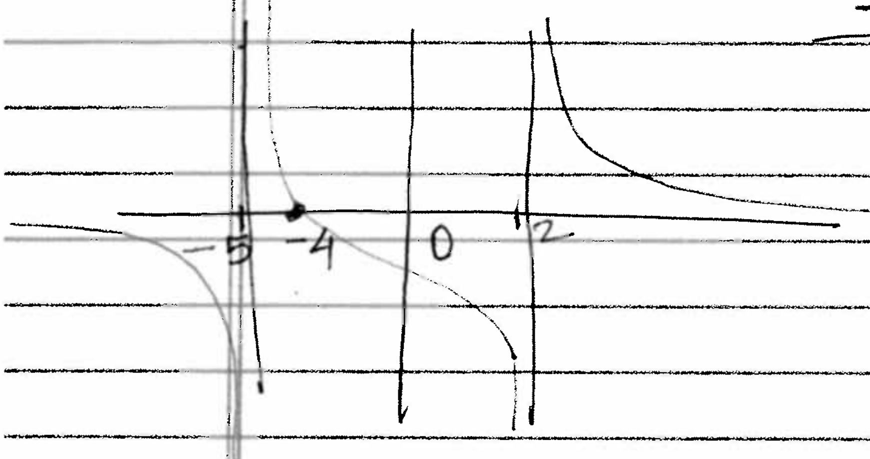
$y=0$.

VA: $x=2$, $x=-5$.

Zeros: $x=-4$.

Signs:



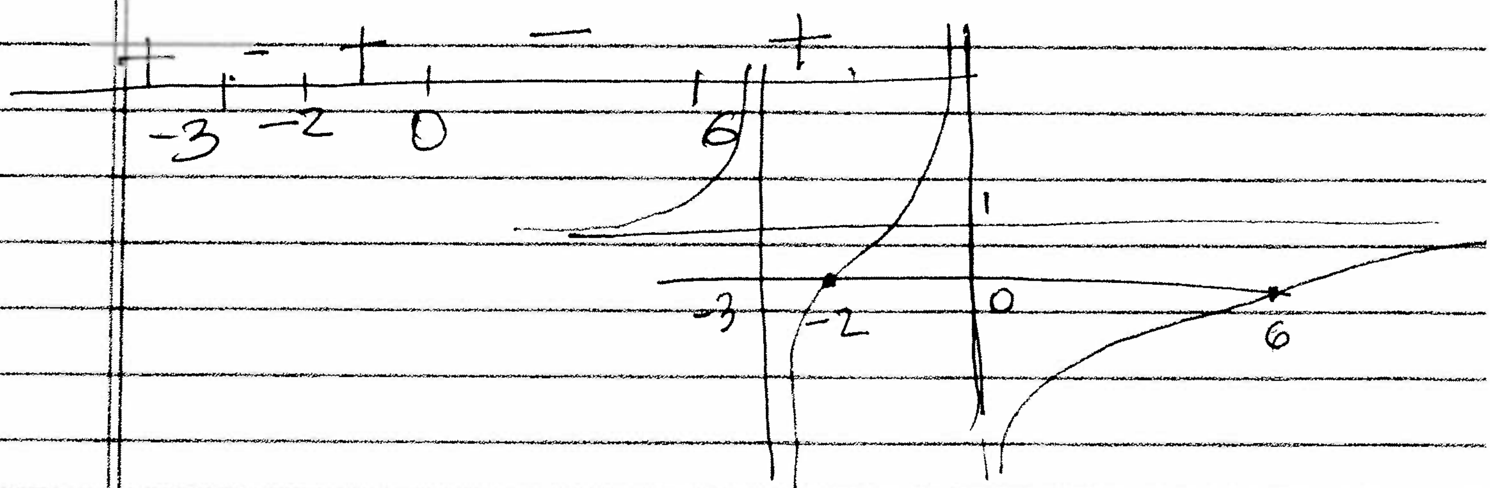


(iii) $$\frac{x^2-4x-12}{x^2+3x} = \frac{(x-6)(x+2)}{x(x+3)}$$

domain $= \{x \mid x \neq 0, -3\}$.

HA: $x^2/x^2 = 1$, $y=1$. VA: $x=0$, $x=-3$.

Zeros: $+6, -2$.



(iv) $\dfrac{x^3-9x}{(x+1)(x-5)(x+6)} = \dfrac{\cancel{x(x^2-9)}\ x(x+3)(x-3)}{(x+1)(x-5)(x+6)}$

domain: $\{x / x \neq -1, 5, -6\}$

HA: $x^3/x^3 = 1$, $y=1$

VA: $x = -1, 5, -6$

zeros: $x = 0, 3, -3$

signs: + (−6) − (−3) + (−1) − (0) + (3) − (5) +