

Classwork 7
Precalculus MTH 130
Instructor: Abhijit Champanerkar

Name: SOLUTIONS

1. Let $f(x) = \frac{2x^2 - 18}{x^2 - 4x - 5} = \frac{2(x-3)(x+3)}{(x-5)(x+1)}$

a. Find the vertical and horizontal asymptotes. □

VA: $x = 5, x = -1$

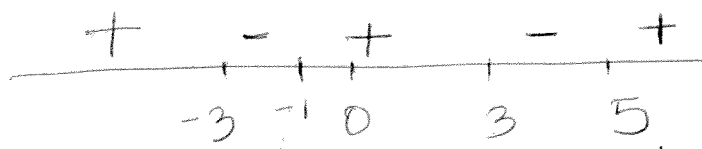
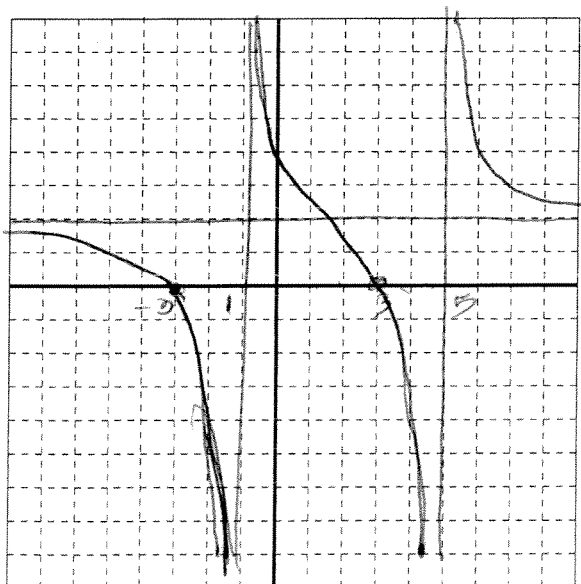
HA: $\frac{2x^2}{x^2} = 2 \Rightarrow y = 2$ HA.

b. Find the intercepts. □

x-intercept $f(x) = 0 \quad x = -3, 3$

y-intercept $= f(0) = 18/5$

c. Using test points and the above information, sketch the graph of $y = f(x)$. □



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1. Fill in the following table.

θ	$0 = 0^\circ$	$\pi/6 = 30^\circ$	$\pi/4 = 45^\circ$	$\pi/3 = 60^\circ$	$\pi/2 = 90^\circ$	$\pi = 180^\circ$
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
$\tan \theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0

2. Convert from degree to radians: (a) $480^\circ = \underline{8\pi/3}$ (b) $540^\circ = \underline{3\pi}$

3. Convert from radians to degrees: (a) $4\pi/3 = \underline{240^\circ}$ (b) $7\pi/6 = \underline{210^\circ}$

4. Find the reference angle and the terminal point on the unit circle determined by the real numbers:

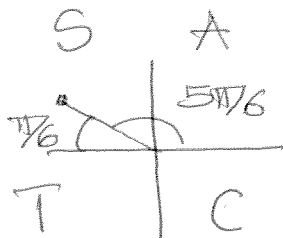
(a) $t = -\pi/4$



$$r = \pi/4$$

$$P = (\sqrt{2}/2, -\sqrt{2}/2)$$

(b) $s = 5\pi/6$



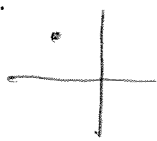
$$P = (-\cos \pi/6, \sin \pi/6)$$

$$= (-\sqrt{3}/2, 1/2)$$

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1. Find $P(x, y)$ on the unit circle if P is in the second quadrant and the y-coordinate is $3/7$.



$$y = \frac{3}{7}, \quad x^2 + y^2 = 1 \Rightarrow x^2 + \frac{9}{49} = 1, \quad x^2 = 1 - \frac{9}{49} = \frac{40}{49}$$

$$x = \pm \frac{\sqrt{40}}{7}$$

2nd quad $\Rightarrow x = -\frac{\sqrt{40}}{7}$

$$P = (-\frac{\sqrt{40}}{7}, \frac{3}{7})$$

2. If $\cos t = 1/3$ and t is in quadrant IV, find the values of all the trigonometric functions at t .



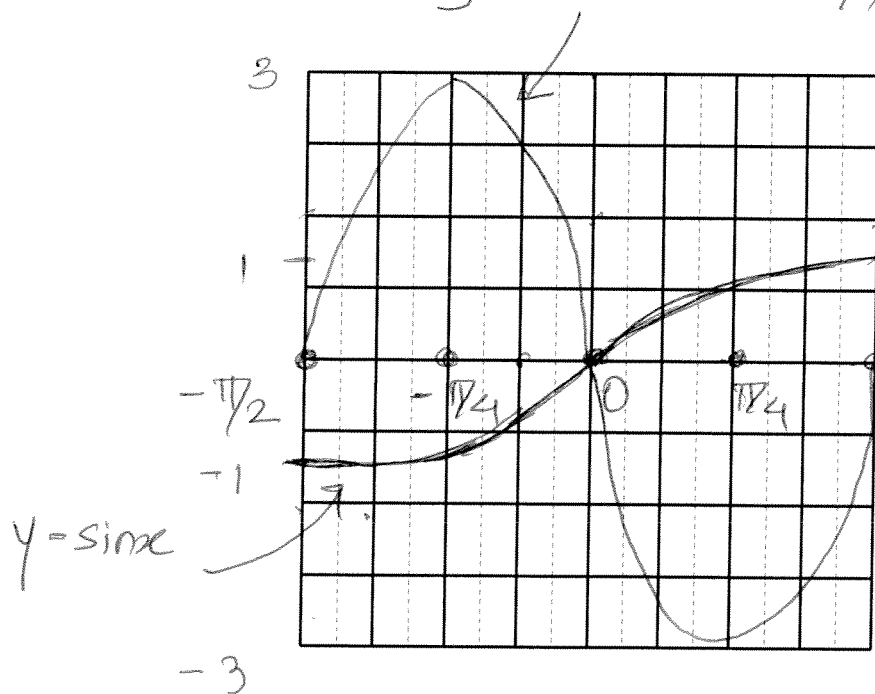
$$\cos^2 t + \sin^2 t = 1$$

$$\frac{1}{9} + \sin^2 t = 1 \Rightarrow \sin^2 t = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\sin t = \pm \frac{\sqrt{8}}{3}$$

4th quad $\Rightarrow \sin t = -\frac{\sqrt{8}}{3}, \quad P = (\frac{1}{3}, -\frac{\sqrt{8}}{3})$

3. Plot the graph $y = \sin x$ and $y = 3 \sin(2x + \pi)$ on the same graph below. Mark the points on the x-axis clearly.



$$y = 3 \sin 2(x - (-\pi/2))$$

$$\text{amp} = 3$$

$$k = 2$$

$$\text{period} = \frac{2\pi}{2} = \pi$$

$$\text{phase shift} = -\pi/2$$

$$0 \quad \pi/2 \quad \pi \quad 3\pi/2 \quad 2\pi$$

$$0 \quad \pi/4 \quad \pi/2 \quad 3\pi/4 \quad \pi$$

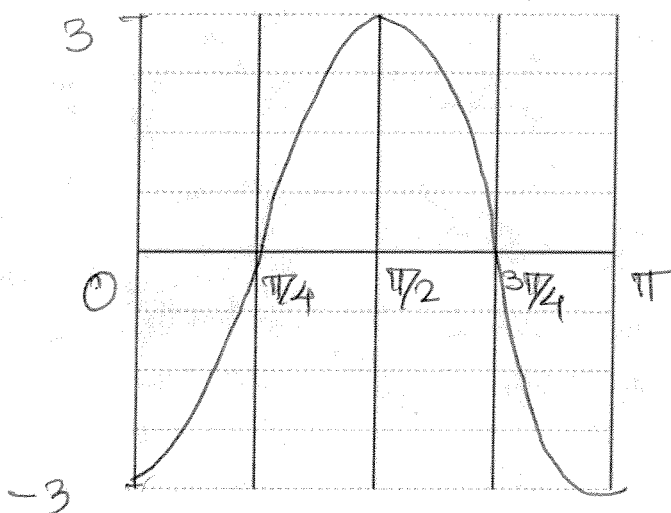
$$-\pi/2 \quad -\pi/4 \quad 0 \quad \pi/4 \quad \pi/2$$

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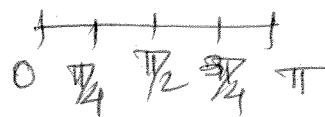
Find the amplitude, period, frequency and phase shift for the following functions. Plot them on the given grid by carefully choosing the scale on both the axes.

1. $y = -3 \cos 2x$

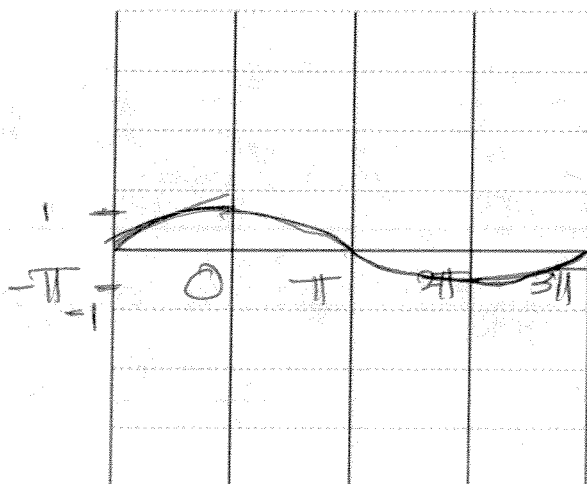


$k=2$

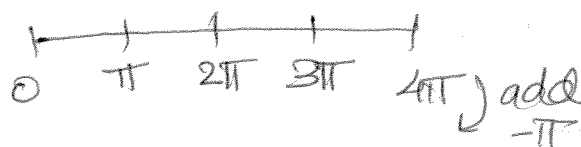
Amplitude = 3
 Period = $2\pi/2 = \pi$
 Frequency = $1/\pi$
 Phase shift = 0



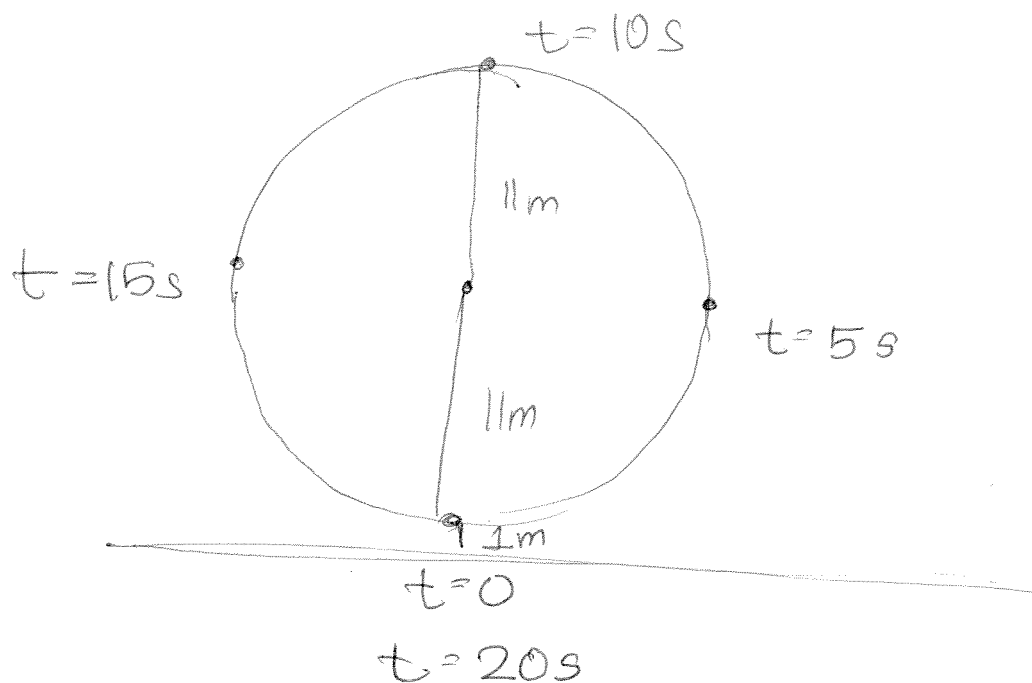
2. $y = \sin \frac{1}{2}(x + \pi)$



Amplitude = 1
 Period = $2\pi/1/2 = 4\pi$
 Frequency = $1/4\pi$
 Phase shift = $-\pi$



3. A ferris wheel has a radius of $11m$ and the bottom of the wheel passes $1m$ above the ground. If the ferris wheel makes one complete revolution every $20s$, find an equation that gives the height above the ground of a person on the ferris wheel as a function of time, assuming at $t = 0$ person starts at bottom (i.e. height $1m$). (Hint: draw pictures).



amp = $11m$,

period = time taken to
complete one revolution
 $= \frac{2\pi}{K} = 20s$

$t=0$, height = $1m$,
starts at bottom.

$\Rightarrow K = \frac{2\pi}{20} = \frac{\pi}{10}$.

$$y = -11 \cos\left(\frac{\pi}{10}t\right) + 12$$

check: $t=0$, $y=1m$ ✓

$t=10$, $y=23m$. ✓

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1. Convert from degree to radians: (a) 480° (b) 540° (c) 315° □

$$480^\circ = 8\pi/3 \text{ rad}, \quad 540^\circ = 3\pi \text{ rad}$$

$$315^\circ = \frac{7\pi}{4} \text{ rad}$$

2. Convert from radians to degrees: (a) $4\pi/3$ (b) $7\pi/6$ □

$$\frac{4\pi}{3} = 240^\circ, \quad \frac{7\pi}{6} = 210^\circ$$

3. Find area of a sector of a circle with central angle 60° and radius 3 mi. □



$$\begin{aligned} \text{Area} &= \frac{1}{2} \theta r^2 = \frac{1}{2} \times \frac{\pi}{3} \times 3 \\ &= \frac{\pi}{2} \text{ mi}^2 \\ \theta \text{ rad} &= 60^\circ = \pi/3 \end{aligned}$$

4. A ceiling fan with 16 in blades rotates at 45 rpm. Find the angular speed of the fan in rad/min and the linear speed of the tip of the blade in in/min. □

$$\omega = 45 \text{ rpm} \Rightarrow \omega = 45 \times 2\pi \text{ rad/min} = 90\pi \text{ rad/min}$$

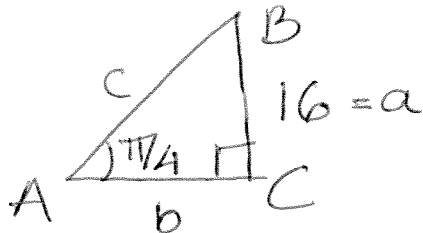
$$r = 16 \text{ in}$$

$$\begin{aligned} v &= r\omega = 16 \times 90\pi \text{ in/min} \\ &= \underline{1440\pi \text{ in/min}} \end{aligned}$$

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1. Solve the right triangle $\triangle ABC$ with $\angle C = \pi/2$, $\angle A = \pi/4$ and $|BC| = 16$. Draw a picture. □



$$\angle B = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

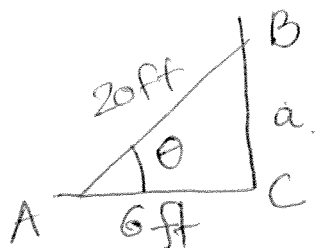
$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b} = \frac{16}{b}$$

$$\tan \frac{\pi}{4} = 1 = \frac{16}{b} \Rightarrow b = 16$$

$$c^2 = a^2 + b^2 = 16^2 + 16^2 = 256 + 256 = 512$$

$$c = \sqrt{512} = 16\sqrt{2} = 22.63$$

2. A 20-ft ladder leans against a building so that the base of the ladder is 6-ft from the base of the building. What is the angle of elevation of the ladder? How high does the ladder reach on the building? Draw pictures. □



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{20}$$

$$\theta \text{ is such that } \cos \theta = \frac{6}{20}$$

$$\theta = \cos^{-1} 6/20 = 1.27 \text{ rad}$$

$$\tan \theta = \frac{a}{6}, \quad a = 6 \tan(1.27)$$

$$a = 19.08$$

~~The~~ ladder goes up to 19.08 ft.
angle of elevation = 1.27 rad.