Review for Test 1
MTH 102
Mathematical Excursions

1. Solving first-degree equations

a) $7x + 6 = 14$ b) $6x - 3 = 4x + 1$ c) $3(x + 2) = 7(x - 2)$ d) $4 = 3(x - 3) + 4 - 2x$

2. Central University reports that 4677 men and 4982 women were enrolled as undergraduates. It also reports that 867 faculty members were employed full-time. What is the student-faculty ratio at Central University?

3 Cindy sold 9 tickets to the school play and Sarah sold 12 tickets. What is the ratio of the number of tickets Cindy sold to the number of tickets Sarah sold?

4 Which is the more economical purchase, a 12-ounce container of yogurt priced at $2.29 or a 5-ounce container of yogurt priced at $0.89?

5 Bigtown University reports 10,555 male undergraduates, 14,742 female undergraduates, and 5128 faculty members. Calculate the student-faculty ratio at Bigtown University. Write the ratio using the word *to*.

6 Out of 630 doctors surveyed, 504 recommended aspirin therapy to their patients. Write the ratio, as a fraction in simplest form, of doctors who recommended aspirin therapy to the total number of doctors surveyed.

7 The velocity of an object is given by the equation $v = 125 - 32t$, where v is the velocity in feet per second and t is the number of seconds after the object is released. How fast will the object be moving 3.2 seconds after it is released?

8 Which is $S = C + rC$ solved for r?

9 Solve $h = \frac{At}{b_1 + b_2}$ for A

10 The cost of a calling card telephone call is \$2.00 for the first five minutes and \$0.75 for each minute over five minutes. Find the length of a call that cost \$8.75.

11 Solve.
$\frac{4}{6} = \frac{7}{x}$

12 Solve.
$\frac{4}{x} = \frac{5}{3}$

13 A consultant earns \$113,000 per year by working 200 days per year. Assuming her daily salary is the same, how much would her annual income be if she worked 250 days in one year?

14 If you travel 348 miles in your car on 12 gallons of gasoline, how far can you travel in your car on 8 gallons of gasoline?

15 A waitress earns \$18,600 per year by working only during the 20 weeks of the peak season. Assuming her weekly salary is the same, how much would her annual income be if she worked 50 weeks in one year?

16 Write 0.052 as a percent.

17 Write 7.3% as a decimal.

18 Write 1.56 as a percent.

19 Write $\frac{3}{25}$ as a percent.

20 Write 24% as a fraction.

21 Write $\frac{3}{8}$ as a percent.

An investor received a dividend of \$28.50, which was 0.2% of the value of the investment. Find the value of the investment.

23 If you answer 32 questions correctly on a 40-question exam, what percent of the questions did you answer correctly?

24 43% of the students are from out-of-state. If there are 18,300 students, how many are from out-of-state?

25 At the start of the spring, it was estimated that 2637 fish were in the lake. At the end of the winter, a new survey was taken, and it was estimated that 3335 fish were in the lake. What is the percent increase in the population to the nearest percent?

26 How many square feet of tile are needed to cover a kitchen floor that is 16 feet long by 11 feet wide?

27 You want to glue a border along the edges of a pennant that has sides measuring 45 cm, 45 cm, and 30 cm. Find the length of border needed.

28 A circular carpet has a diameter of 7 feet. What is the area of the carpet? Round to the nearest tenth.

29 Find the length of baseboard molding needed to edge the bottom of the walls of a square room that is 13 feet long.

Multiple Choice Questions

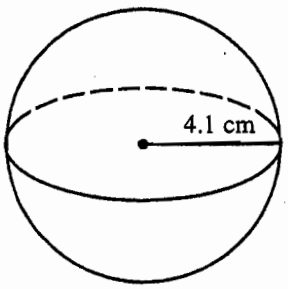
30 What is the volume of a rectangular prism with length 10.5 inches, width 7 inches, and height 4.2 inches?

a. 21.7 cubic inches
b. 73.5 cubic inches
c. 102.9 cubic inches
d. 308.7 cubic inches

31 The radius of the base of a cone is 6 inches. The height of the cone is 7 inches. What is the volume of the cone, to the nearest hundredth?

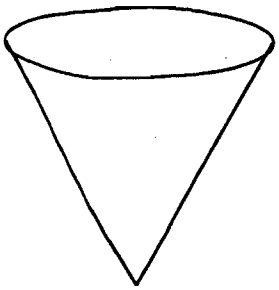
a. 43.98 cubic inches
b. 131.95 cubic inches
c. 263.89 cubic inches
d. 791.68 cubic inches

32 Find the volume of the sphere. Use $3.14 = \pi$.



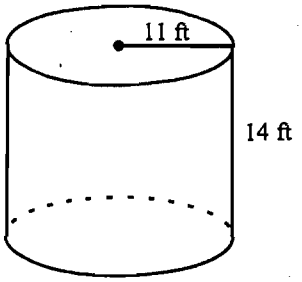
A. 162 cm^3 B. 70 cm^3 C. 302 cm^3 D. 289 cm^3

33 What is the name of the solid shown below?



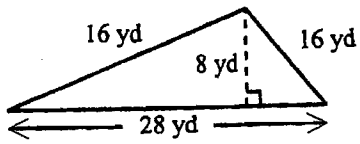
A. sphere B. cone C. cylinder D. triangular pyramid

34 Find the volume of the cylinder. Use $3.14 = \pi$.



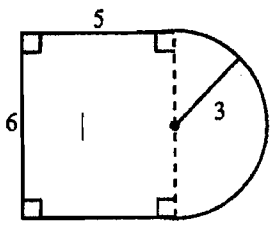
A. 6769.84 ft^3 B. 5319.16 ft^3 C. 1694 ft^3 D. 483.56 ft^3

35 Find the area.



A. 112 yd^2 B. 224 yd^2 C. 24 yd^2 D. 128 yd^2

36 Find the area of the figure. Dimensions are in meters. Use $3.14 = \pi$.

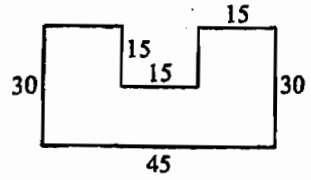


A. 44.13 m^2 B. 34.71 m^2 C. 58.26 m^2 D. 28.26 m^2

37 The length of a rectangular carpet is 6 feet more than its width. If the area of the carpet is 91 square feet, find its length.

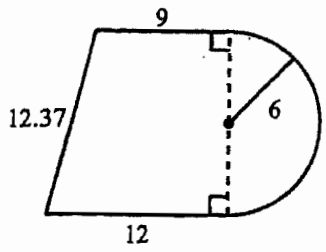
A. 13 ft B. 15 ft C. 7 ft D. 12 ft

38 Find the perimeter of this figure. Dimensions are in feet. All angles are right angles.



A. 90 ft B. 180 ft C. 150 ft D. 195 ft

39 Which is the perimeter of this figure? Dimensions are in yards. Use 3.14 for π.



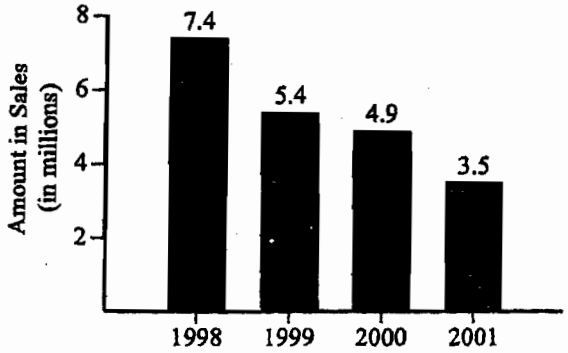
A. 45.37 yd B. 71.05 yd C. 52.21 yd D. 64.21 yd

40 A wooden fence is to be built around a 32- by 44-meter lot. How many meters of fencing will be needed? If the wood for the fence costs $39.00 per meter, what will the wood for the fence cost?

A. 1408 m; $5928.00 B. 1408 m; $54,912.00

C. 152 m; $54,912.00 D. 152 m; $5928.00

41 The graph shows the amount reported in sales for a software company over a four year period. Find the percent of decrease in sales from 1999 to 2000.



A. 110.2% B. 10.2% C. 90.7% D. 9.3%

42 The sales of Kids sneakers rose from $3 million to $3.7 million. Find the percent increase to the nearest whole percent.

A. 2.5% B: 25% C. 2.3% D. 23%

43 An investor received a dividend of $43.95, which was 0.5% of the value of the investment. What was the value of the investment?

a. $87.90
b. $219.75
c. $8790.00
d. $2197.50

44 If you answered 14 questions correctly on a 20-question exam, what percent of the questions did you answer correctly?

a. 28%
b. 43%
c. 65%
d. 70%

45 A study finds that 67% of students prefer internet resources to textbooks. Based on this survey, out of 200 students, how many prefer internet resources to textbooks?

a. 134
b. 67
c. 66
d. 33

46. Theresa made 35 of 40 free throws at basketball practice. Which percent did she make?

A. 12.5% B. 46.7% C. 87.5% D. 5%

47 Margarette correctly answered 36 questions on a English test. She received a score of 90%. How many questions were on the test?

A. 50 B. 45 C. 47 D. 40

48 Which shows 50% as a decimal?

A. 5 B. 0.5 C. 0.05 D. 0.005

49 Which shows 0.96 as a percent?

A. 0.96% B. 9.6% C. 96% D. 960%

50 Which shows $\frac{1}{5}$ as a percent?

A. 0.5% B. 0.2% C. 20% D. 5%

51 Which is 4% written as a fraction?

A. 25 B. $\frac{1}{25}$ C. 4 D. $\frac{2}{5}$

52 An astronaut who weighs 175 pounds on Earth would weigh 45 pounds on the Theta Space Station. If a piece of equipment weighed 2100 pounds on Earth, what would it weigh on the Theta Space Station?

A. 585 lb B. 552 lb C. 540 lb D. 594 lb

53 What is the solution to the proportion $\frac{x}{14} = \frac{6}{5}$?

a. $x = 2.1$
b. $x = 5.9$
c. $x = 11.7$
d. $x = 16.8$

54 The ratio of cars to people in New Zealand is 350 to 1000. Compare as a ratio in simplest form.

A. $\frac{7}{10}$ B. $\frac{350}{1000}$ C. $\frac{7}{40}$ D. $\frac{7}{20}$

55 Find the ratio of 2 hours to 20 minutes.

A. $\frac{10}{1}$ B. $\frac{1}{6}$ C. $\frac{1}{10}$ D. $\frac{6}{1}$

56 Over the course of two hockey seasons, a star hockey player scored 45 goals while playing in 77 games. Which of the following expresses his scoring rate as a unit rate rounded to the thousandths place?

A. 0.316 goals per game
B. 0.416 goals per game
C. 0.584 goals per game
D. 0.484 goals per game

57 Patricia paid $315 for 3 nights at a hotel. What was the nightly rate for her room?

A. $945 per night B. $105 per night C. $210 per night D. $53 per night

58 A writer was paid $6000 for a 2000-word article. Find the unit rate.

A. $3.00 per word
B. $3.33 per word
C. $30.00 per word
D. $0.33 per word

59 A teacher earns $620 for working a 40-hour week. What is the teacher's hourly rate of pay?

a. $12.40 per hour
b. $15.50 per hour
c. $16.00 per hour
d. $20.25 per hour

60 Which is the most economical purchase?

a. 10 ounces of orange juice for $1.29
b. 22 ounces of orange juice for $1.99
c. 48 ounces of orange juice for $2.25
d. 64 ounces of orange juice for $3.29

61 Bob wants to find the best buy on laundry soap. Brand A costs $6.02 for 40 ounces. He has a coupon for $0.50 off the regular price of Brand B, which costs $7.22 for 48 ounces. Brand C costs $5.13 for 32 ounces. Which of these is the best buy?

A. Brand B
B. Brand B and Brand A cost the same per ounce
C. Brand A
D. Brand C

62 Which is $V = \frac{1}{3}s^2h$ solved for h?

a. $h = \frac{1}{3}s^2V$

b. $h = \frac{V}{3s^2}$

c. $h = \frac{3V}{s^2}$

d. $h = 3Vs^2$

63 Solve the equation $A = \frac{1}{2}h(b+c)$ for c.

A. $c = \frac{2}{A}h + b$ B. $c = \frac{2}{h}A - b$ C. $c = \frac{A}{b+c}$ D. $c = \frac{h}{2}A - c$

64 Starlight Tree Farm sells Douglas firs and noble firs. One December they sold 179 more Douglas firs than noble firs. The total number of trees sold was 499. Which equation could be used to solve for n, the number of noble fir trees sold?

A. $2n + 179 = 499$ B. $2n - 179 = 499$

C. $n - 179 = 499$ D. $n + 179 = 499$

65 The formula $p = \frac{1}{20}s + 200$ is used to calculate a shoe salesperson's weekly earnings. In the formula, p represents the total earnings for the week and s represents the total weekly shoe sales. Suppose the employee sold \$1800 worth of shoes in one week. Find the salesperson's weekly earnings.

66 The charge for mailing a fourth-class package through the U.S. Postal Service is $C = 0.08x + 2.58$

where C is the charge in dollars and x is the weight of the package in pounds.

a. Find the charge to mail a package that weighs 9 pounds.

b. How many pounds can be mailed for \$3.54?

A. a. \$3.30
b. 12 lb

B. a. \$0.56
b. 14 lb

C. a. \$3.09
b. 11 lb

D. a. \$3.30
b. 13 lb

Review for Test 2
MTH 102
Mathematical Excursions

Math 102 Review for Test 2

1 If it is equally likely that a child will be born a boy or a girl, use a tree diagram to list the sample space for a family of four.

2 A caterer offers 2 choices of salad, 3 choices of vegetable, and 4 choices of a main dish. How many different dinner combinations are possible?

3 Nine swimmers are at a swim meet. If there are awards for those finishing first, second, and third, how many possible ways can the awards be given?

4 A certain plant will have white flowers if the plant inherits the recessive gene for white flowers from both parents. Otherwise it will have pink flowers. Using P for the dominant pink allele and p for the recessive white allele, suppose one parent plant is Pp and the other is pp. What is the probability that the offspring will have white flowers?

5 Suppose you are choosing a 4-digit personal access code. This code is made up of 2 numbers (1-9 which can be repeated), followed by 2 letters (A-Z which also can be repeated). Find the total number of possibilities.

6 A fair coin is tossed 3 times. What is the probability that two tails and one heads are tossed?

7 Suppose the odds against a certain greyhound winning a race are 6 to 1. What is the probability of the greyhound winning the race?

8 If the probability of winning a certain contest is 0.21, what is the probability of not winning the contest?

9 What is the probability of drawing three cards in succession (without replacement) from a regular deck of playing cards and having them all be face cards?

The table below shows the preferred mode of transportation for individuals in a particular town by age group. Use the data in the table to solve problems 10 – 12

Age	Car	Bus	Train
18–25	147	221	107
26–34	230	194	76
35–49	218	130	87
50+	156	211	45

10 If a person is randomly chosen from the town's population, what is the probability that the person is aged 35–49 or prefers bus transportation?

11 If a person is randomly chosen from the town's population, what is the probability that the person does not prefer train transportation?

12 If a person is randomly chosen from the town's population, what is the probability that the person prefers car transportation, given that he or she is aged 35 or older?

13 Find the median for the data in the following list: 9, 14, 3, 7, 25, 10, 18

14 Eight students in a history class received test grades of 94, 81, 79, 85, 83, 92, 86, and 88. Find the mean of these test scores.

15 An instructor determines a student's weighted average from quizzes, tests, and a final. Each test counts as two quizzes, and the final counts as six quizzes. Emily has quiz scores of 98, 92, 89, 97, 100, and 95. Her test scores are 99, 95, and 98. Her final score is 91. Find Emily's weighted mean for the course.

16 Find the mode for the data in the following list: 7, 14, 22, 8, 15, 13, 4, 15, 9, 12

17 The following numbers were obtained by sampling a population.
3, 5, 10, 16, 8, 19, 3, 14, 22, 10
Find the standard deviation of the sample.

18 Find the range of the data in the following list: 5, 9, 23, 11, 7, 18, 9, 1

19 A soda machine dispenses soda into 24-ounce cups. Tests show that the actual amount of soda dispensed is normally distributed, with a mean of 22.8 ounces and a standard deviation of 0.6 ounce. If a cup is chosen at random, what is the probability that the machine will overflow the cup?

20 Find the variance for the population sample given in question 6.

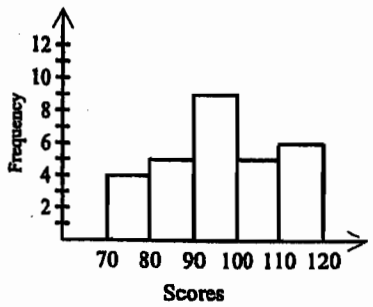
Multiple Choice Questions

21 The life expectancy of a fluorescent tube is normally distributed with a mean of 6000 hours and a standard deviation of 500 hours. Find the probability that a tube lasts for more than 6500 hours. Use the Empirical Rule.

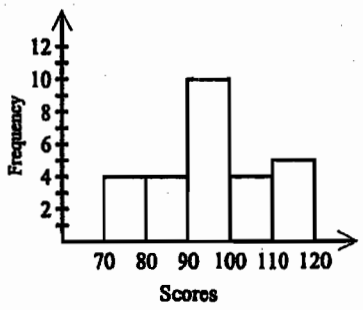
A. 0.3410 B. 0.6590 C. 0.8410 D. 0.1590

22 The golf scores for the 26 members of the Belmont Country Club were 82, 79, 95, 113, 106, 98, 117, 71, 104, 83, 75, 114, 106, 99, 82, 88, 91, 117, 107, 104, 91, 82, 96, 93, 99, 98. Make a histogram using ten-point intervals that show the frequency distribution of the scores.

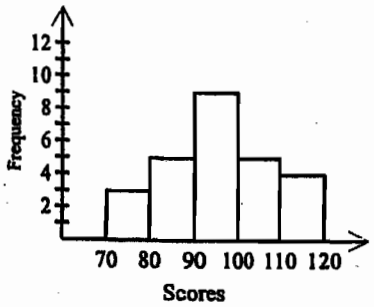
A.



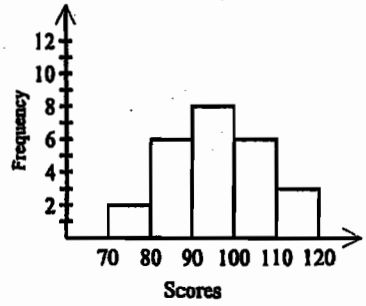
B.



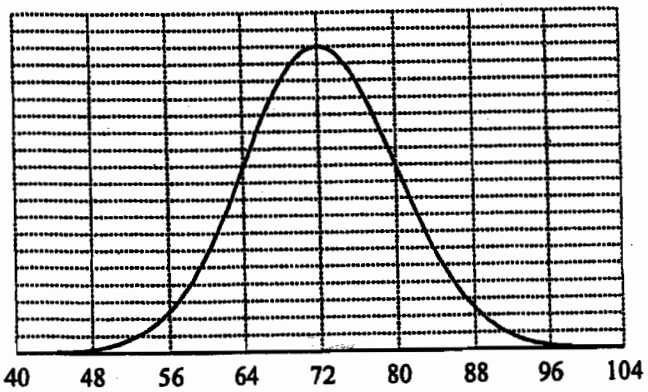
C.



D.



23 Below is a graph of the distribution of the scores of 15,000 students on a standardized eleventh-grade algebra readiness test.



a. This distribution is approximately normal. Estimate the mean, , and standard deviation of this distribution.

b. About how many of the 15,000 students had scores between 56 and 80? Explain your reasoning. Use the Empirical Rule.

24 Find the mean and the standard deviation of the given data sample. Round to the nearest hundredth if necessary.
11, 4, 19, 6, 22, 24, 20, 21, 26

A. Mean = 20, standard deviation ≈ 8.6

B. Mean = 17, standard deviation ≈ 8.6

C. Mean = 17, standard deviation ≈ 7.98

D. Mean = 20, standard deviation ≈ 7.98

25 Six separate blood tests revealed that a patient had total blood cholesterol levels of 239, 234, 226, and 241. What is the mean of the blood cholesterol tests?

a. 235 b. 236
c. 237 d. 238

26 What is the median for the data in the following list?
81, 60, 78, 34, 6, 73, 56, 10, 59

a. 34 b. 56
c. 59 d. 60

27 What is the mode for the data in the following list?
3, 2, 8, 2, 16, 7, 9, 2, 5, 11, 7

a. 2 b. 7
c. 16 d. Both (a) and (b)

28 An instructor determines a student's weighted average from assignments, tests, and a final. Each test is worth two assignments, and the final is worth eight assignments. Charmian has assignment scores of 72, 65, 61, 59, 78, and 67. Her test scores are 98, 100, and 95. Her final score is 99. What is Charmian's weighted mean for the course?

a. 79.4 b. 84.7
c. 89 d. 93.1

29 What is the range of the data in the following list?
5, 44, 13, 9, 6, 5, 32, 49, 28

a. 5 b. 13
c. 39 d. 44

30 The following numbers were obtained by sampling a population.
6, 4, 3, 6, 1, 2, 5, 9, 10, 4
What is the standard deviation of the sample?

a. 2.7 b. 2.9
c. 5 d. 7.4

31 What is the variance for the sample given in question 6?

a. 7.3 b. 7.4
c. 8.2 d. 8.4

32 So far in art class, a student's quiz scores are 89%, 88%, 81%, and 76%. What score does the student need on the fifth quiz to have a mean quiz score of 85%? All the quizzes have equal weights.

A. 92% B. 87% C. 83.5% D. 91%

33 How many 3-digit numbers can be formed, without replacement, from the digits 3, 5, 7 and 9?

a. 24
b. 81
c. 256
d. 27

34 A card is chosen from each of the four suits in a standard deck of playing cards. How many elements are in the event that a 7, 8, 9 and 10 are chosen?

a. 1
b. 13
c. 52
d. 24

35 What is $C(12, 7)$?

a. 3,991,680
b. 95,040
c. 120
d. 792

36 How many different letter arrangements are possible using all the letters of the word *mathematics*?

a. 88
b. 55,440
c. 86,400
d. 4,989,600

37 There are 8 items on a birthday wish list. How many different ways can 3 items be chosen as presents?

a. 56
b. 336
c. 6720
d. 120

38 A card is selected at random from a standard deck of playing cards. Compute the probability that the card is a heart.

a. $\frac{1}{4}$
b. $\frac{1}{3}$
c. $\frac{1}{9}$
d. $\frac{3}{4}$

39 The height of a certain plant is determined by a dominant allele T corresponding to tall plants, and a recessive allele t corresponding to short plants. A genotype of tt corresponds with a short plant, and all other genotypes correspond with a tall plant. If one parent has genotype TT and the other has genotype Tt, what is the probability that the offspring will be tall?

a. $\frac{1}{4}$
b. $\frac{3}{4}$
c. $\frac{1}{2}$
d. 1

40 If two fair dice are rolled one time, what are the odds in favor of rolling a sum of 10?

a. 11 to 1
b. 1 to 11
c. 12 to 1
d. 1 to 12

41 A box contains four red marbles, six green marbles, and two blue marbles. What is the probability of pulling two green marbles followed by a blue marble if the marbles are pulled from the bag without replacement?

a. $\frac{1}{22}$
b. $\frac{5}{144}$
c. $\frac{127}{110}$
d. $\frac{1}{24}$

42 If the probability it will rain tomorrow is 0.35, what is the probability that it will not rain tomorrow?

a. 0.1225
b. 0.35
c. 0.2275
d. 0.65

43 If two fair dice are rolled one time, what is the probability that the sum on the two dice is an odd number or a number less than 3?

a. $\frac{1}{2}$
b. $\frac{19}{36}$
c. $\frac{4}{9}$
d. $\frac{17}{36}$

Use the data in the table below which shows favorite subjects of students in a high school by grade.

Age	Math	English	Science
10^{th}	35	47	38
11^{th}	22	89	64
12^{th}	37	65	53

44 If a student is randomly chosen, what is the probability that the student is in 10^{th} grade or prefers English?

a. 0.71
b. 0.10
c. 0.61
d. 0.23

45 If a student is randomly chosen, what is the probability that the student is in 11^{th} grade and prefers Math?

a. 0.13
b. 0.05
c. 0.23
d. 0.60

46 A spinner is numbered from 1 through 9. What is the probability of spinning a number less than 3 or greater than 7 in a single spin?

A. $\frac{5}{9}$ B. $\frac{7}{9}$ C. $\frac{4}{9}$ D. $\frac{2}{3}$

47 A bag contains 9 green marbles, 8 red marbles, and 7 blue marbles. Without looking, a marble is drawn from the bag. What is the probability that a red or a blue marble will be drawn?

A. $\frac{7}{72}$ B. $\frac{7}{69}$ C. $\frac{1}{23}$ D. $\frac{5}{8}$

48 The probability of an event is $\frac{4}{9}$. Find the odds *against* this event occurring.

A. 5 to 4 B. 5 to 9 C. 4 to 9 D. 4 to 5

49 In an elementary school class, the probability that a student is taller than five feet is $\frac{4}{7}$. Find the odds in favor of a student chosen at random from the class being taller than five feet.

A. 4 to 7 B. 3 to 7 C. 4 to 3 D. 3 to 4

50 The odds *against* a particular candidate winning an election are estimated to be 6 to 5. If those odds are accurate, what is the probability that that candidate will win the election?

A. $\frac{5}{6}$ B. $\frac{6}{5}$ C. $\frac{5}{11}$ D. $\frac{6}{11}$

51 If the probability of rain this afternoon is 0.49, what is the probability that it will not rain this afternoon?

A. 0.49 B. 0.25 C. 0.76 D. 0.51

52 The table shows the drink preferences of 50 shoppers at a mall.

Drink Survey

Drink	Number of Shoppers
A	5
B	12
C	14
D	11
E	8

If one of the 50 shoppers surveyed is chosen at random, what is the probability that the shopper preferred either drink A or drink B?

A. $\frac{17}{50}$ B. $\frac{6}{25}$ C. $\frac{11}{25}$ D. $\frac{3}{125}$

53 A bag of marbles contains 7 blue, 7 green, 8 red, 9 yellow, and 3 black marbles. If you reach in the bag and draw one marble at random, what is the probability that you will draw a yellow marble?

A. $\frac{25}{34}$ B. $\frac{9}{25}$ C. $\frac{9}{34}$ D. $\frac{1}{9}$

54 Suppose you mix up the cards below and choose one without looking. What is the probability of selecting a consonant?

A. 1 B. $2\frac{1}{2}$ C. $\frac{5}{7}$ D. $\frac{2}{7}$

55 A coin is tossed and a die is rolled. What is the probability that the coin shows tails and the die shows 6?

A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{12}$ D. $\frac{1}{4}$

56 A spinner is evenly divided into 9 equal areas and numbered from 1 through 9. What is the probability of spinning a number less than 7 on a single spin?

A. $\frac{1}{3}$ B. $\frac{7}{9}$ C. $\frac{2}{3}$ D. $\frac{2}{9}$

57 How many different arrangements can be made using all of the letters in the word ORANGE, if each letter is used exactly once?

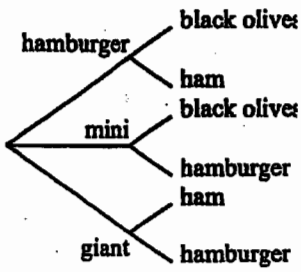
A. 15 B. 6 C. 720 D. 156

58 Suppose you are choosing a wall color from among 6 different paint colors, and also choosing an accent color from among 6 different paint colors and 3 different levels of shine (flat, semi-gloss, and glossy). How many choices do you have?

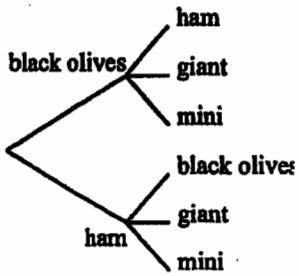
A. 111 B. 109 C. 108 D. 36

59 At a pizza parlor, Lu has a choice of pizza toppings and sizes. The topping choices are black olives, ham, and hamburger. The size choices are giant and mini. Which tree diagram shows the number of possible single-topping pizzas that Lu can order?

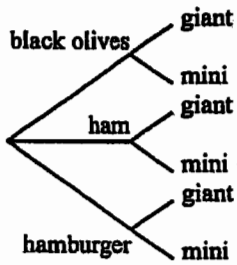
A.



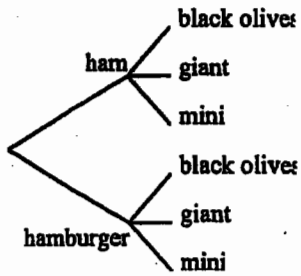
B.



C.



D.



60 List the elements of the sample space defined by selecting an even date from the possible dates in June.

A. {2, 4, 6, 8, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30}

B. {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29}

C. {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30}

D. {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28}

REVIEW FOR TEST #3

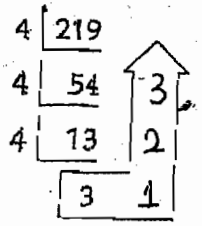
Module A: Numeration Systems

The contents of this document will help, both instructors and the students who wish to incorporate a supplemental modulo, proposed by Mr. E.Naseem for Math for Liberal Arts (a.k.a Mth 102), in reviewing the contents of the material/s. The said modulo comprise of three strands.

Strand I:

Converting from Base Ten to Another Base

The most efficient method of converting a number written in base ten to another base makes use of a *successive division process*. For example, to convert 219 to base four, divide 219 by 4 and write the quotient 54 and the remainder 3, as shown below. Now divide the quotient 54 by the base to get a new quotient of 13 and a new remainder of 2. Continuing the process, divide the quotient 13 by 4 to get a new quotient of 3 and a remainder of 1. Because our last quotient, 3, is less than the base, 4, we stop the division process. The answer is given by the last quotient, 3, and the remainders, shown in red in the following diagram. That is, 219 = 3123 four



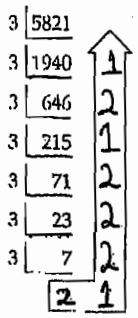
You can understand how the successive division process converts a base ten numeral to another base by analyzing the process. The first division shows there are 54 fours in 219, with 3 ones left over. The second division shows that there are 13 sixteens (two successive divisions by 4 is the same as dividing by 16) in 219, and the remainder 2 indicates that there are 2 fours left over. The last division shows that there are 3 sixty-fours (three successive divisions by 4 is the same as dividing by 64) in 219, and the remainder 1 indicates that there is 1 sixteen left over. In mathematical notation these results are written as follows.

$$219 = (3x64)+(1x16)+(2x4)+(3x1)=(3x4^3)+(1x4^2)+(2x4^1)+(3x4^0)=3123_{four}$$

Provided below are two more examples to clarify the system/method.

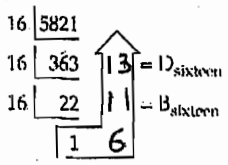
Convert 5821 to a. base three and b. base sixteen.

a.



$5821 = (21222121)_3$

b.



$5821 = (16BD)_{16}$

Bear in mind most commonly used conversion is Base-2 Conversion, commonly referred to as Binary Code. In these modern times of computers such conversions have enabled us to convert massive numerical data in to efficient and applicable codes. Provided below is a table that will show us the conversion of hexadecimals into binary equivalents.

Hexadecimal and
Binary
Equivalents

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

For further review and practice:

Text: Mathematical Excursions, Chapter 4, Page 213 Review Exercises,

Questions 33, 34, 75 and 77
Test Page 244/ 11 & 12 a

Strand II:
Modular Arithmetic:
The main purpose is to familiarize the students with the basic arithmetic operations such as addition, subtraction and multiplication and enable them to see the importace of Remainder in arithmetic. As the concepts mature, we will try to prove the validity of various congurnce models and eventually solve basic congurance equations.

Solve a Congruence Equation Solve: $2x + 1 = 3 \bmod 10$
Solution
Beginning with 0, substitute each whole number less than 10 into the congruence equation.

$x = 0$	$2(0) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 1$	$2(1) + 1 \equiv 3 \bmod 10$	A solution
$x = 2$	$2(2) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 3$	$2(3) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 4$	$2(4) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 5$	$2(5) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 6$	$2(6) + 1 \not\equiv 3 \bmod 10$	A solution
$x = 7$	$2(7) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 8$	$2(8) + 1 \not\equiv 3 \bmod 10$	Not a solution
$x = 9$	$2(9) + 1 \not\equiv 3 \bmod 10$	Not a solution

Having found a solution such as shown in example above, we can find more solutions by simply adding the given modular value (1.0 in this example)
For example 1+10 = 11
8+10 = 18----------

Further practice: Text: Mathematical Excursions, Chapter 7,Page 446 Review Excersices.

Questions 11, 14, 15 and 27.

Further practice: Text: Mathematical Excursions, Chapter 7, page 446, Review Exercises.
Questions 11, 14, 15 & 27

Strand III
Applications of Modular Arithmetic:

a. **Determine a Check Digit (ISBN CODE, International Standard Book Number)**

Determine the ISBN check digit for the book *A Brief History of Time* by Stephen Hawking. The first nine digits of the ISBN are 0-553-05340- i.

Solution

Use the ISBN congruence equation.

$0(10) + 5(9) + 5(8) + 3(7) + 0(6) + 5(5) + 3(4) + 4(3) + 0(2) + x \equiv 0 \bmod 11$ $155 + x \equiv 0 \bmod 11$

To solve the congruence equation, try whole number values of x that are less than the modulus. Because $155 + 10 = 165 \equiv 0 \bmod 11$, the check digit is 10. Recall that a check digit of 10 is recorded as an X. Therefore, the ISBN is 0-553-05340-X.

b. **Determine the Check Digit for a UPC**

A new product a company has developed has been assigned the UPC 0-21443-32912-..L Determine the check digit for the UPC.

Solution

Use the UPC congruence equation.

$$0(3) + 2(1) + 1(3) + 4(1) + 4(3) + 3(1) + 3(3) + 2(1) + 9(3) + 1(1) + 2(3) + x = 0 \bmod 10$$

$$69 + x = 0 \bmod 10$$

To solve the congruence equation, try whole number values of x that are less than the modulus. Because $69 + 1 = 0 \bmod 10$, the check digit is 1.

c. **Determine a Valid Credit Card Number**

Credit card numbers are normally 13 to 16 digits long. The first one to four digits are used to identify the card issuer. The table below shows the identification prefixes used by four popular card issuers.

Credit Cards	Prefix	Number of digits
MasterCard	51 to 55	16
Visa	4	13 or 16
American Express	34 or 37	15
Discover	6011	16

.The Luhn algorithm, used to determine whether a credit card number is valid, is calculated as follows. Beginning with the next-to-last digit (the last digit is the check digit) and reading from right to left, double every other digit. If a digit becomes a two-digit number after being doubled, treat the number as two individual digits. Now find the sum of the new list of digits; the final sum must equal 0 mod 10. The Luhn algorithm is demonstrated in the next example.

Determine whether 5234 8213 3410 1298 is a valid credit card number.

Solution

Highlight every other digit, beginning with the next-to-last digit and reading from right to left.

523482133 4 1 0 1 298

Next double each of the highlighted digits.

10 2 6 4 16 2 2 3 6 4 2 0 2 2 18 8

Finally, add all digits, treating two-digit numbers as two single digits.

$$(I + 0) + 2 + 6 + 4 + (I + 6) + 2 + 2 + 3 + 6 + 4 + 2 + 0 + 2 + 2 + (I + 8) + 8 = 60$$

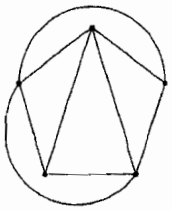
Further practice: Text: Mathematial Excursions, Chapter 7, page 446, Review Exercises.

Questions 33, 35 and 37.

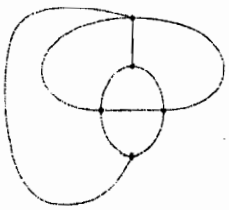
REVIEW FOR TEST #3

Module B Graph Theory

Use the graphs below for exercises 1 and 2.



Graph A

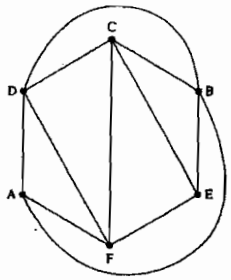


Graph B

1. Determine whether the graphs A and B are equivalent.

2. Is graph A connected? Is graph A a complete graph?

Use the graph below for exercises 3 and 4.



3. Determine whether the graph is Eulerian. If it is, find an Euler circuit. If it is not, explain how you know.

4. Does the graph have an Euler walk? If so, find one. If not, explain why not.

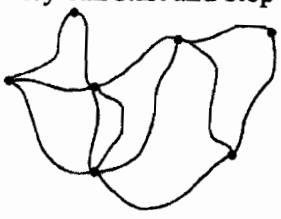
Use the table below for questions 5 and 6. An X in the table represents an intersection of the two roads.

	Park St.	Center St.	Frank Ave.	Main St.	Dock Rd.
Park St.	--	X	X	X	X
Center St.	X	--			
Frank Ave.	X		--	X	
Main St.	X		X	--	X
Dock Rd.	X			X	--

5. Draw a graph to represent the information in the table.

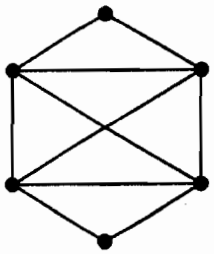
6. Is it possible to plan a drive that traverses each road and returns to the starting point without traveling any road twice?

7. A county road crew is going to repaint the centerline stripes on the roads shown below. Can the crew do the job by traveling each section of road just once, assuming they can start and stop at any intersection?

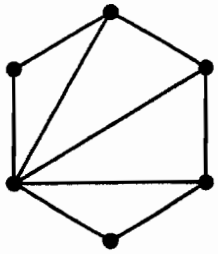


8 Identify the graph that contains an Euler walk.

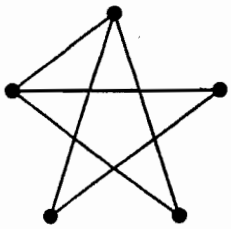
A.



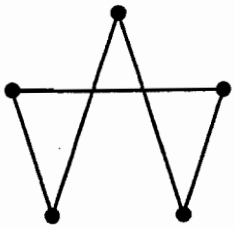
B.



C.



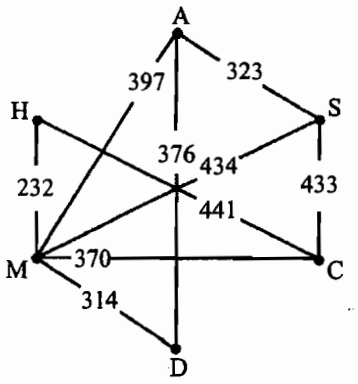
D.



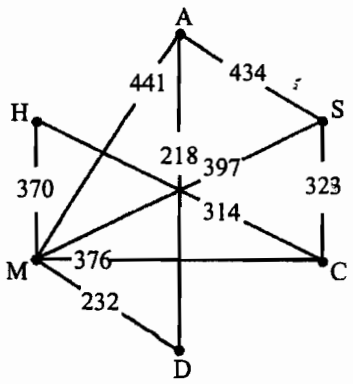
9. A representative of a large corporation needs to travel to six cities to see major clients who have concerns about recent cost increases for the products they use. None of the trips were planned in advance, so the representative must pay the most recently published one-way airfares. Due to special offers, some fares are lower than might be expected compared to others. All the fares are indicated in the table below. Draw a weighted graph to represent the situation shown in the table.

City	Atlanta	Seattle	Chicago	Denver	Miami	Houston
Atlanta	—	370	376	—	323	232
Seattle	370	—	434	218	356	—
Chicago	376	434	—	—	441	433
Denver	—	218	—	—	397	—
Miami	323	356	441	397	—	314
Houston	232	—	433	—	314	—

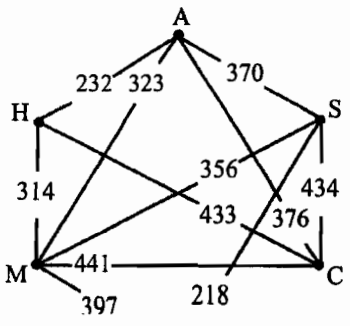
A.



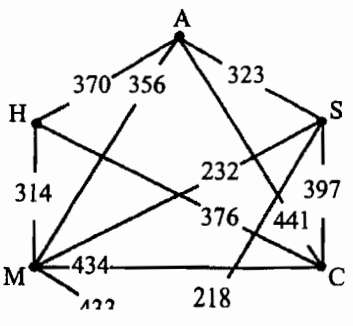
B.



C.



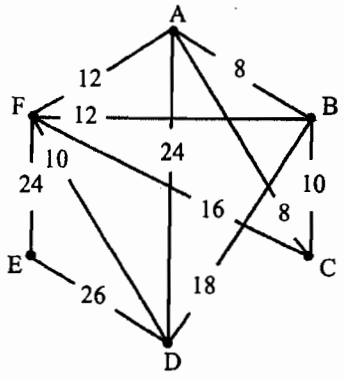
D.



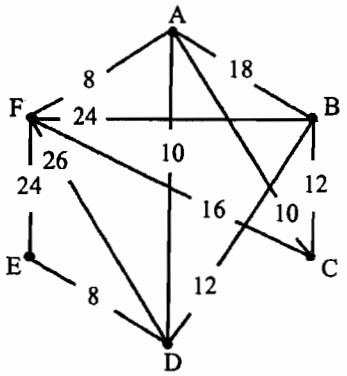
10. A produce delivery service makes daily deliveries to independent grocery stores. The table below lists the distances (in miles) between the stores, measured along the available direct routes. Draw a weighted graph to represent the situation shown in the table.

Store	A	B	C	D	E	F
A	—	8	8	24	—	12
B	8	—	10	18	—	12
C	8	10	—	—	—	16
D	24	18	—	—	26	10
E	—	—	—	26	—	24
F	12	12	16	10	24	—

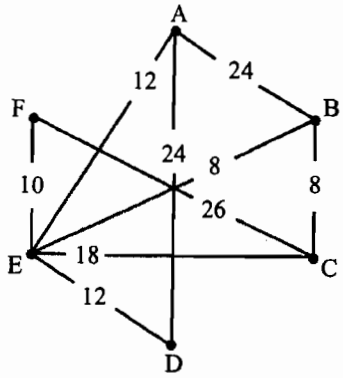
A.



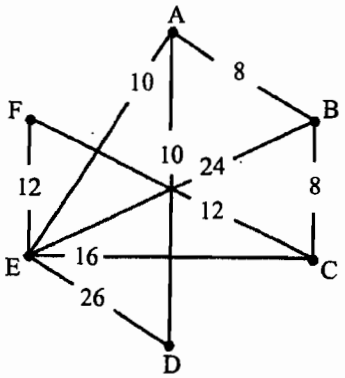
B.



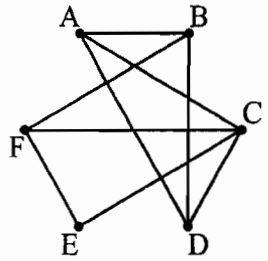
C.



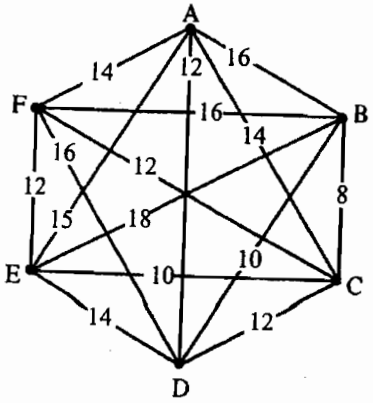
D.



11. The CEO of a small regional airline wants to visit the airline's operations centers at each of the airports where the airline has terminals. In the graph below, an edge indicates that the airline has a direct flight between the corresponding airports. Flying only on the airline's own routes, is it possible for the CEO to visit every office exactly once and return to the same airport at which the trip originated?



12. Use the Greedy Algorithm to find a Hamiltonian circuit in the weighted graph shown below. Start at vertex A.



13 A contest winner's prize is a free round-trip airline ticket to any city in Europe, with an unlimited stayover. While there, the contestant wants to travel to five other cities before returning to the original airport for the return flight home. The table below shows the one-way airfares between the cities. Beginning with A, find a sequence of cities that can be followed to give the lowest total airfare and give that airfare. Use the Edge-Picking Algorithm.

City	A	B	C	D	E	F
A	—	—	$378	$357	$259	$369
B	—	—	$447	$257	$301	$394
C	$378	$447	—	$289	$387	$432
D	$357	$257	$289	—	$298	$263
E	$259	$301	$387	$298	—	$358
F	$369	$394	$432	$263	$358	—

14. A small school district has contracted with a local produce supplier to make daily deliveries to each school in the district. The table below lists the distances (in miles) between the schools, measured along the available direct routes. School A is located closest to the supplier's produce market. Starting at that school, use the Greedy Algorithm to find a route that takes the delivery van to each school once and returns to school A. Give the total mileage for that route.

School	A	B	C	D	E	F
A	—	11	9	9	18	13
B	11	—	10	8	7	15
C	9	10	—	8	14	7
D	9	8	8	—	—	8
E	18	7	14	—	—	14
F	13	15	7	8	14	—

A. A-C-F-B-D-E-A; 57 mi

B. A-C-F-D-B-E-A; 57 mi

C. A-C-F-D-B-E-A; 55 mi

D. A-C-F-B-D-E-A; 55 mi

REVIEW FOR TEST #3

Math 102 - Module C: Voting Methods - Review

1. In an election with four candidates, in how many ways may a preference ballot be made out?

2. In choosing a winner in an election with four candidates using the pairwise comparison method, how many head-to-head mathups have to be analyzed?,

3. A family is planning a reunion and has narrowed down the choice to four cities: New York (*N*), Chicago (*C*), Atlanta (*A*) and San Francisco (*S*). The family members are asked to rank the cities in order of preference. The preference schedule is shown below.

	Number of Voters				
Rank	7	3	12	8	13
First	*C*	*N*	*A*	*S*	*N*
Second	*S*	*C*	*N*	*A*	*A*
Third	*N*	*S*	*C*	*N*	*S*
Fourth	*A*	*A*	*S*	*C*	*C*

a) How many people voted?
b) Does any city have a majority of the votes?
c) Determine the winning city using the plurality method.
d) Determine the winning city using the plurality with elimination method.
e) Determine the winning city using the Borda Count method.
f) Determine the winning city using the pairwise comparison method.

4. There are three candidates for president of a large organization: *A*, *B* and *C*. A vote is taken resulting in the following preference schedule.

	Number of Voters		
Rank	2691	2416	237
First	*A*	*B*	*C*
Second	*C*	*C*	*B*
Third	*B*	*A*	*A*

a) Use the Borda Count method to determine the winner.
b) Show that the majority criterion has been violated.
c) Identify a candidate who wins all head-to-head matchups.
d) Explain why the Condorcet criterion has been violated.
e) If *B* drops out of the race, determine the winner of the new election using the Borda Count method.
f) Explain why the independence of irrelevant alternatives has been violated.

5. Consider the following preference schedule.

	Number of Voters			
Rank	7	5	4	1
First	*A*	*C*	*B*	*D*
Second	*D*	*A*	*C*	*B*
Third	*B*	*B*	*D*	*A*
Fourth	*C*	*D*	*A*	*C*

a) Determine the winner using the pairwise comparison method.
b) Suppose the election is rerun, but the one voter on the right decides to change his vote and makes *A* his second choice and *B* his third choice. Who wins the election now?
c) Explain why the monotonicity criterion has been violated.

6. Consider the following preference schedule.

	Number of Voters				
Rank	14	4	10	1	8
First	*A*	*B*	*C*	*C*	*D*
Second	*B*	*D*	*B*	*D*	*C*
Third	*C*	*C*	*D*	*B*	*B*
Fourth	*D*	*A*	*A*	*A*	*A*

a) Who wins using the plurality with elimination method?
b) Is there a Condorcet candidate?
c) What Fairness Criterion has been violated?

7. The 45 members of a school's football team vote on three nominees, *A*, *B*, and *C*, by approval voting for the award of "most improved player" as indicated in the following table. An X indicates an approval vote. (Note that two players "abstained," voted for none of the nominees, and one player voted for all of the nominees.)

	Number of Voters							
Nominees	7	8	9	9	6	3	1	2
A	X			X	X		X	
B		X		X		X	X	
C			X		X	X	X	

a) Which of the nominees is selected for the award?
b) Which of the nominees is announced as the runner-up for the award?