Time and Place TBA

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Textbook Required: ELEMENTARY DIFFERENTIAL GEOMETRY, by *Andrew Pressley.*

Also suggested:
DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES, by *Manfredo P. do Carmo.*
MODERN DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES, by *Alfred Gray.*

Course Outline This course is an introduction to differential geometry, an important subject of modern mathematics. It will primarily be focused on the study of curves and surfaces in three dimensional spaces. We will see how the notion of derivative introduced in multi-variable calculus can naturally be extended to differentiate functions defined on curves and surfaces. This generalization combined with elements of linear algebra allow to introduce several notions that allow to measure how much a curve or surface is bent in the Euclidean space. One goal of this course is to find invariants for curves and surfaces, giving criteria to see if two surfaces can be deformed one into the other. We will go across very nice and interesting theorems, for example the Theorema Egregium which explains why there cannot be an accurate planar map of the world.

More concretely we plan to cover the following topics:

1. Curves in $\mathbb{R}^3$: arclength, curvature, torsion, Fundamental Theorem of space curves.
2. Surfaces in $\mathbb{R}^3$: regular and parametrized surfaces, first fundamental form, Gauss map, second fundamental form, principal curvatures, Gaussian curvature, mean curvature.
3. Intrinsic Geometry of Surface: (local) isometries, vector fields, covariant derivative, Theorema Egregium, Fundamental Theorems for surfaces, geodesics.
4. Gauss-Bonnet Theorem: geodesic curvature, local Gauss-Bonnet Theorem, topology and triangulations of surfaces, global Gauss-Bonnet Theorem.

Prerequisites Students need to be comfortable with vector calculus and linear algebra.
Official prerequisites for the course are:
(Mth 233 and Mth 330) or (Mth 233, Mth 334 and Mth 338).

Course Grade The final course grade is determined as follows:

Homework	10%
Midterms	25% + 25%
Exams	Final 40%

Lesson Plans Detailed lesson plans are as follows:

Lesson	Sections	Topics
1	1.1, 1.2, 1.3	Parametrized curves, arc-length
2	1.4, 1.5	Local theory of curves
3	1.6, 1.7	Global theory of curves
4	2.1, 2.2	Regular surfaces
5, 6	2.3	Functions on parametrized surfaces
7	2.4	Differential of a map
8	2.5	First fundamental form
9	2.6	Surface area, orientation
10	none	Review
11	none	Exam One
12, 13	3.1, 3.2	Gauss map and its properties
14	3.3	Gauss map in local coordinates
15	3.4	Vector fields
16	3.5	Minimal surfaces
17	4.1, 4.2	Conformal maps and Isometries
18	4.3	Gauss theorem, review
19	none	Exam Two
20	4.4	Parallel transport and geodesics
21	4.5	Gauss-Bonnet theorem
22	4.6	Exponential map
23	5.1, 5.2	The rigidity of the sphere
24	5.3	Theorem of Hopf-Rinow
25	5.4	Variations of the arc length
26	5.5	Jacobi fields and conjugate points
27	5.6	Covering space and the Hadamard theorem
28	none	Review